1:11 Using Properties and Relationships

Teacher Notes

Math Milestones™

Central math concepts

A major development in adding and subtracting during grade 1 is the progression from Level 1 methods that directly model the quantities and operations involved to Level 2 methods that involve counting-on and then to Level 3 methods that use properties of operations and relationships between operations to replace a given problem with an easier problem. See the figure, from the *Progression* document,[†] p. 6, and the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20," pp. 14–17.

> **Methods used for solving single-digit addition and subtraction problems**
>
> *Level 1. Direct Modeling by Counting All or Taking Away.* Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.
>
> *Level 2. Counting On.* Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.
>
> For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).
>
> *Level 3. Convert to an Easier Problem.* Decompose an addend and compose a part with another addend.

An important example of a Level 3 strategy is the make-a-ten method for single-digit addends with sum greater than 10. For example, to use the make-ten strategy to calculate the sum 8 + 5 in task 1:11, students can think of the sequence of calculations

$$8 + 5 = 8 + 2 + 3 = 10 + 3 = 13.$$

The strategy requires providing the number (2) that makes 10 when added to 8, and providing the number (3) that makes 5 when added to 2. In the last step, the strategy leverages understanding of the meaning of teen numbers. The *Progression* document (p. 16) summarizes these "three prerequisites that reach back into kindergarten":[‡]

a) knowing the partner that makes 10 for any number (CCSS K.OA.A.4 sets the stage for this),

b) knowing all decompositions for any number below 10 (CCSS K.OA.A.3 sets the stage for this), and

c) knowing all teen numbers as 10 + n (e.g., 12 = 10 + 2, 15 = 10 + 5, see CCSS K.NBT.A.1 and 1.NBT.B.2b).

The problem 7 + 4 = 10 + ? in task 1:11 is a sort of snapshot of the make-a-ten method in progress. The equation 7 + 4 = 10 + 1 can be interpreted as a result of decomposing 4 = 3 + 1.

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ____	8 − ____ = 2
13 − 4 = ____	____ − 5 = 4
7 + 4 = 10 + ____	6 + ____ = 12

Answer

Left column: 13, 9, 1. *Right column*: 6, 9, 6. Student explanations may vary (see "Central math concepts").

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.B; MP.1, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Procedural skill and fluency

Make-a-ten methods are Level 3 because they use properties of addition to replace a given problem with an easier problem. Make-a-ten methods also take advantage of place value, because of the role of 10 in the strategy.

An example of a Level 2 method would be finding the difference 13 - 9 by counting on. Paraphrasing the *Progression* document (p. 14), this approach involves seeing the 9 as part of 13, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9: thus the student may say,

"Niiiiine, ten, eleven, twelve, thirteen."
1 2 3 4

Students keep track of how many they counted on (here, 4) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Niiiiine…") is natural and indicates that the student differentiates between the first addend and the counts for the second addend. Counting up from 9 to subtract is preferable than counting down from 13 to subtract, both because counting up is easier and also because counting up reinforces the relationship between addition and subtraction.

Compared to 13 - 9, the problem 13 - 4 in task 1:11 is perhaps harder to solve using Level 2 methods. A possible Level 3 method for 13 - 4 would be to think that

$$\begin{aligned} 13 - 4 &= 13 - 3 - 1 \\ &= 10 - 1 \\ &= 9. \end{aligned}$$

Alternatively, a different Level 3 method would be to think of 13 - 4 as an unknown addend problem, 4 + ☐ = 13, and then think that since 4 + 6 = 10,

$$4 + 6 + 3 = 13$$

$$4 + 9 = 13$$

So that 13 - 4 = 9.

Rewriting a subtraction equation in a different form can also be helpful for the problem 8 - ☐ = 2 in task 1:11. Equations of this form can arise when analyzing word problems with situation type "Take From with Change Unknown" (see Teacher Notes for task **1:13 Falling Icicles**). Thinking of 8 as decomposed into two parts, one part equal to 2, the equation 8 - ☐ = 2 asks for the other part—a problem that can be restated as 2 + ☐ = 8. Now the unknown addend could be found by a Level 2 method, or simply thought of as 8 - 2, that is, 6. Similarly, the problem ☐ - 5 = 4 in task 1:11 could be rewritten as 4 + 5 = ☐, which leads to the answer 9.

Finally, another way to find a difference by using the relationship between addition and subtraction is simply to know the relevant decomposition. For example, in the problem 6 + ☐ = 12 from task 1:11, some or many students by end of grade 1 might simply remember the "doubles fact" 6 + 6 = 12 and thus quickly know that the unknown addend is 6. Remembering all the single-digit sums and fluently finding the related differences isn't a grade 1 expectation (CCSS 2.OA.B.2), but some or many grade 1 students will gain enough experiences during the year to know a few, or more than a few decompositions from memory by the end of the year. If students do find a difference $C - A$ by remembering the related sum $A + B = C$, they don't need to explain or justify the answer beyond simply "I knew that $A + B = C$." But they should be able to discuss and understand another

Additional notes on the design of the task

- In the left-hand column, the last problem features an addition sign on both sides of the equal sign. This emphasizes number sense and the properties of addition as well as the meaning of the equal sign (CCSS 1.OA.D.7).
- The task includes the direction, "Tell how you got the answers" so that students can produce mathematical language, and so as to reveal the mix of Level 1, Level 2, and Level 3 methods at work in students' thinking.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:11? In what specific ways do they differ from 1:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

student's Level 2 or Level 3 method as the occasion to do so arises. No student needs to use all the methods described in the *Progression* document, but the methods all have mathematical value, and students should be able to discuss and understand each other's methods.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting on; finding partners of 10 and partners of smaller numbers; using the structure of teen numbers; and using the relationship between addition and subtraction.

Extending the task

How might students drive the conversation further?

- Students could choose one of the equations in task 1:11 and create a word problem that corresponds to it.
- If a student and a partner used different methods for one of the problems in task 1:11, the student and partner could switch and try each other's method.

Related Math Milestones tasks

1:9

1:9 Write the missing numbers.

4 + 5 = ____ 7 − 4 = ____
10 − 8 = ____ 2 + 6 = ____
4 + ____ = 10 7 + ____ = 10

Task **1:9 Fluency within 10** concentrates on problems within the grade 1 fluency goal (CCSS 1.OA.C.6). The word problems in grade 1 involve addition and subtraction within 20; see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

2:5

2:5 Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2 4✓

Click here for student handout 2:5

2:8

2:8 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 2:8

In later grades, tasks **2:5 Sums of Single-Digit Numbers** and **2:8 Fluency within the Addition Table** represent the culminating fluency and remembering goals for addition and subtraction within 20 (CCSS 2.OA.B.2).

K:5

K:5 [Teacher puts 3 red counters on table.] **Put some blue counters here to make 10 counters in all.** [Student completes this task.] **How many counters did you add?** [Student determines the answer.]

Write the missing number: 3 + ____ = 10

K:8

K:8 [Teacher holds out 5 paper clips.] **How many do I have?** [Student counts the paper clips.] [Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?** [Student counts the paper clips.]

How many are in my other hand?

K:12

K:12 **Draw 16 circles. Use a** [favorite color] **marker for 10 of them. Use a pencil for the rest.** [Student draws.]

How many are [favorite color]**? How many are in pencil?**

Write the missing number: 16 = 10 + ____

K:13

K:13 Write or say the missing numbers.

3 + 1 = ____ 2 + 3 = ____
5 + 0 = ____ 2 − 2 = ____
4 − 3 = ____ 5 − 3 = ____

In earlier grades, task **K:5 Adding to Make a Group of Ten** involves finding a partner of 10. Task **K:8 Five Behind the Back** involves decompositions of 5, and task **K:12 Make Ten and Some More** involves the structure of teen numbers. Task **K:13 Fluency within Five** concentrates on problems within the kindergarten fluency goal (CCSS K.OA.A.5).

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in this Teaching Note refer to this *Progression* document.

‡ See also "How I see addition facts," (Zimba, 2016).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?