1:13 Falling Icicles

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

1:13

When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Answer

5 icicles fell while I slept.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

Additional notes on the design of the task

The photo in the task shows 8 icicles, which allows students to discuss the situation in relation to the photo. The photo also clarifies the context for students who may be unfamiliar with cold weather.

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation in task 1:13 can be thought of as "Take From with Change Unknown."[‡] Some icicles were 'taken away,' but initially it is unknown how many icicles were taken away. Thus, an equation model for the situation could be written as 8 - ☐ = 5. This situation equation (p. 13) could then be rewritten as a solution equation, 8 - 5 = ☐.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in $A + \square = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding. In terms of the situation in task 1:13, the number of icicles that fell can be found by subtraction, 8 - 3 = 5, and 5 can also be seen as an addend: the number of icicles that would have to 'put back up' in order to restore the original number: 3 + 5 = 8.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: subtracting two one-digit numbers; adding two one-digit numbers; and fundamental concepts of addition and subtraction.

Extending the task

How might students drive the conversation further?

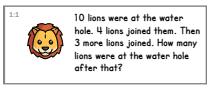
- Students could reinterpret the situation along the lines of tasks **K:2 Two Groups of Books** or **1:6 Two Groups of Straws** (*Put Together/Take Apart with Total Unknown*) by drawing two groups of icicles: one group of 3 icicles hanging by the window, and one group of 5 icicles lying on the ground. This view of the situation may help to relate the equations 8 - 3 = ? and 3 + 5 = 8.
- Students could create their own word problem to represent 8 - 3 = ?.

Curriculum connection

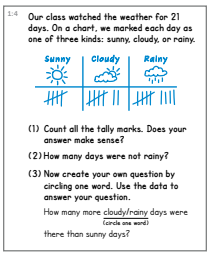
1. In which unit of your curriculum would you expect to find tasks like 1:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:13? In what specific ways do they differ from 1:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

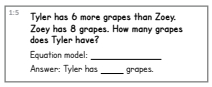
1:1



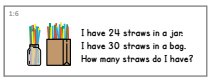
1:4



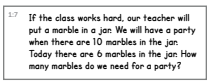
1:5



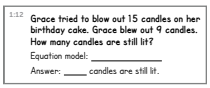
1:6



1:7



1:12



Besides task 1:12, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) *and Compare with Difference Unknown (‘how many more’ language)* (part (3)); **1:5 Tyler’s Grapes**, *Compare with Bigger Quantity Unknown (‘more’ language)*; **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; and **1:12 Blowing Out Candles,** *Put Together/Take Apart with One Addend Unknown.*

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?