1:14 Shape True/False

Teacher Notes

Central math concepts

As explained in the *Progression* document (p. 2),[†] the three themes of elementary-grades geometry are:

- Composing and decomposing shapes;
- Reasoning with shape components, shape properties, and shape categories; and
- Spatial relations and spatial structuring.

These three themes are involved in the three statements that students evaluate in task 1:14.

To verify the first statement, two of the triangles can be composed to make a rectangle, and then the rectangles can be used as a new unit for composing a square, as shown in the figure that appears in the Answer section. Thus, "With experience, students are able to use a composed shape as a new unit in making other shapes" (p. 4). One reason this is important is that "there is suggestive evidence that this type of composition corresponds with, and may support, children's ability to compose and decompose numbers" (p. 3).

The second statement involves shape attributes and categories. The G Progression document (p. 3) describes three *levels of geometric thinking* that describe increasing sophistication with this learning progression:

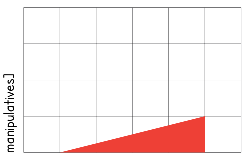
- **Visual/Syncretic level.** Students recognize shapes, for example, a rectangle "looks like a door."
- **Descriptive level.** Students perceive properties of shapes, for example, a rectangle has four sides, all its sides are straight, opposite sides have equal length.
- **Analytic level.** Students characterize shapes by their properties, for example, a rectangle has opposite sides of equal length and four right angles.
- **Abstract level.** Students understand, for example, that a rectangle is a parallelogram because it has all the properties of parallelograms.

Identifying the second statement as false can involve a mix of Syncretic reasoning (the pentagon "doesn't look square") and Descriptive reasoning (even though the square is oriented diagonally, it still has the properties of a square) (p. 8).

A recurrent question in mathematics concerns what it means for two objects of a certain kind to count as "the same."[‡] In later-grades geometry for example, congruence defines what it means for two distinct figures to count as "the same" despite differences of position or orientation. To a young student, a square oriented diagonally looks different from a square with its sides oriented with the edges of

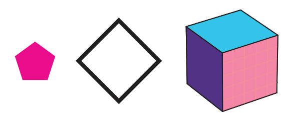
1:14 **One statement below is false. Find the false statement. How did you decide?**

[Student uses manipulatives]

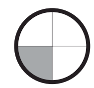


A square can be created using triangles like this one.

None of these are squares.

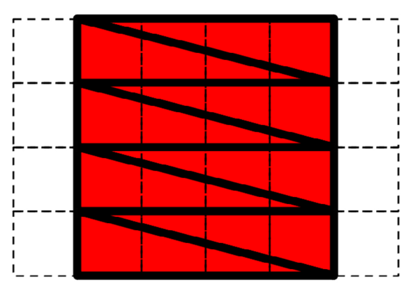


The shaded part of the circle is one fourth of the whole circle.



Answer

The false statement is "None of these are squares." Explanations may vary. Identifying the first statement as true involves composing the triangles to make a square, as shown in the figure. Identifying the second statement as false relies on recognizing that the second shape in the corresponding group of three shapes is, indeed, a square. Identifying the third statement as true relies on recognizing that four of the shaded parts make up 1 whole circle.



Click here for a student-facing version of the task.

the paper—reasonably enough, since with respect to orientation at least, the diagonal square *is* different. Likewise, there is a literal sense in which a red square and a blue square *are* different, at least with respect to color. Students aren't wrong to perceive these differences. But the progression from Visual/Syncretic toward Descriptive involves coming to use the category "square" in mathematical discourse in such a way that orientation, color, and overall size "no longer count."

Levels of thinking in spatial structuring

Levels of thinking portrayed by different students as they attempted to complete a drawing of an array of squares, given one column and row. This was an assessment, not an instructional task.

In the third statement, a circle is spatially structured into congruent parts. Spatial structuring is "the mental operation of constructing an organization or form for an object or set of objects in space," and it builds on students' experiences with shape composition (p. 4). Such spatial structuring will be used in grade 3 to understand area measurement for rectangles and in grade 5 to understand volume measurement for right rectangular prisms. Spatial structuring is also involved in partitioning of wholes during division and fraction reasoning in grades 3–6. Thus, "spatial structuring precedes meaningful mathematical use of the structures" (p. 4). The figure shows levels of spatial structuring portrayed by different students in response to an assessment task (p. 11).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: recognizing and naming shapes; composing triangles and rectangles; and identifying and naming shape attributes.

Refer to the Standards

1.G.A; MP.1, MP.3, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts

Additional notes on the design of the task

The first statement is intended to be explored using manipulatives. The second statement could also be posed using manipulatives (flat pentagon, flat square, and solid cube).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:14? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:14? In what specific ways do they differ from 1:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

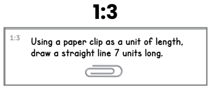
Extending the task

How might students drive the conversation further?

- With reference to the first statement, if students compose a square, they could state the properties that convince them they have in fact made a square instead of a (non-square) rectangle.
- With reference to the second statement, if students correctly explain that the statement is false "because the second shape is a square," they could continue the discussion by explaining why the first and third shapes (the pentagon and the cube) *aren't* squares. Press for precision using attribute language: for example, a sufficient reason why the pentagon isn't a square is that it has five sides, not four. (This is a more robust explanation than simply saying that the pentagon isn't a square "because it's a pentagon.")
- With reference to the third statement, students could compose two of the fourths and describe the resulting shape as half of the circle.

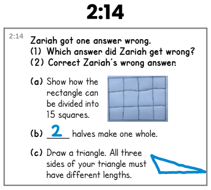
Related Math Milestones tasks

1:3

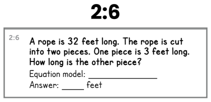


Spatial structuring and composing/decomposing are involved in length measurement (iterating length units), as in task **1:3 Paper Clip Length Units**.

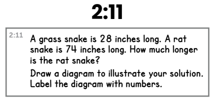
2:14



2:6

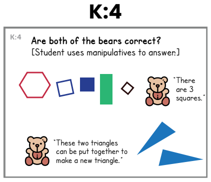


2:11



In later grades, task **2:14 Correcting a Shape Answer** involves the same three themes of elementary grades geometry as task 1:14. Composing and decomposing is involved in adding and subtracting lengths, as in tasks **2:6 Cutting a Rope** and **2:11 Grass Snake vs. Rat Snake**.

K:4



In earlier grades, task **K:4 Bears Talk About Shapes** involves shape attributes and composing shapes.

† Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression*.

‡ In algebra, there are two important questions along these lines: (1) What does it mean for two different-looking expressions to be equivalent? (2) What does it mean for two different-looking functions to be the same function?

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?