1:1 Lions at the Watering Hole

Teacher Notes





Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

- 1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
- 2. Guess at the operation to be used.
- 3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
- 4. Try all the operations and choose the most reasonable answer.
- 5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
- 6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
- 7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

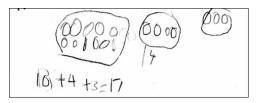
Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word



10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

Answer

There were 17 lions at the water hole after that. While the task does not require students to write an equation or represent the problem in other ways, a sample of student work is shown here.



<u>Click here</u> for a student-facing version of the task.

Refer to the Standards

1.OA.A.2, 1.OA; MP.2, MP.4. Standards codes refer to <u>www.corestandards.</u> org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing baseten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

Multi-step problems like task 1:1 combine situation types. The action in task 1:1 is a sequence of two processes of "Add To with Result Unknown."[‡] In the first of these two processes, there are some lions at the water hole to begin with, and then more lions come, resulting in a new (initially unknown) group size. Then in the second process, still more lions arrive, resulting in a final (initially unknown) group size.

One approach to the problem is to identify the intermediate group size as a quantity worth knowing, and to calculate that quantity as 10 + 4 =14. At that point the final group size can be calculated in another step as 14 + 3 = 17. An alternative strategy could be to combine the two "waves" of lion arrivals, 4 + 3 = 7, and then calculate the final group size as 10 +7 = 17. The fact that the two strategies both work and both lead to the same result could be thought of as reflecting the basic "physics" of the situation in the task, and mathematically the agreement is guaranteed by the associative property of addition:

$$(10 + 4) + 3 = 10 + (4 + 3).$$

Grade 1 students might perform the calculations 10 + 4 and 10 + 7 simply by understanding the structure of the teen numbers (<u>CCSS K.NBT.A.1</u> and <u>1.NBT.B.2b</u>). A calculation of 14 + 3 might proceed by Level 2 or Level 3 methods; see the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20," <u>pp. 14–17</u> in the *Progression* document. An example of a Level 2 method would be counting on from 14; thus, for 14 + 3, a student may say,

"Fourrrteeeen, fifteen, sixteen, seventeen."

1

2

3

Students keep track of how many they counted on (here, 3) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Fourrrteeeen...") is natural and indicates that the student differentiates between the first addend and the counts for the second addend.

Meanwhile, an example of a Level 3 method would be to think that 14 = 10 + 4, so that 14 + 3 = 10 + 4 + 3 = 10 + (4 + 3) = 10 + 7 = 17.

Students for whom the calculations in the task are time-consuming and/or effortful may need to be redirected to the context after obtaining the result 17, so as to relate the numbers in this equation to the context and answer the question in the task.

Additional notes on the design of the task

- The situation types being combined in the task are simple and familiar, to balance the fact that the situations are being combined in a multi-step problem.
- The starting number of 10 creates opportunities to learn and use the meaning of teen numbers.

Curriculum connection

- In which unit of your curriculum would you expect to find tasks like 1:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:1? In what specific ways do they differ from 1:1?
- 2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*



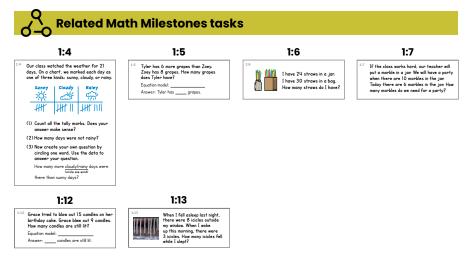
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two one-digit numbers with sum less than 10; writing numbers, expressions, and equations; working with teen numbers; and fundamental concepts of addition.

→ Extending the task

How might students drive the conversation further?

- Students could ask and answer additional questions about the situation. For example,
 - What if the 10 lions that were at the water hole first got scared away—how many lions would be at the water hole after they left?
 - A photographer took a picture that showed 11 of the lions. How many lions were not shown in the picture?
- Students could relate expressions or equations to the situation. For example, in the expression 10 + 4 + 3 what quantity in the situation does the number 10 refer to? (The number 10 refers to "the number of lions that were there first." Note that naming a quantity is different from naming the numerical value of the quantity.)



Besides task 1:1, other word problems and their situation types in grade 1 are as follows: tasks **1:4 Analyzing Weather Data**, *Put Together/Take Apart* with Total Unknown (part (2)) and Compare with Difference Unknown ('how many more' language) (part (3)); **1:5 Tyler's Grapes**, Compare with Bigger Quantity Unknown ('more' language); **1:6 Two Groups of Straws**, Put Together/Take Apart with Total Unknown; **1:7 Class Marble Jar**, Add To with Change Unknown; **1:12 Blowing Out Candles**, Put Together/Take Apart with One Addend Unknown; and **1:13 Falling Icicles**, Take From with Change Unknown.

In earlier and later grades, see the <u>Map of Addition and Subtraction</u> <u>Situations in K-2 Math Milestones</u>.

- † Sowder, Larry. (1988). Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report. San Diego State Univ, CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf
- ‡ For the other situation types, see <u>Table 2, p. 9</u> of Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K-5, Operations and Algebraic Thinking (Common Core Standards Writing Team, May 29, 2011. Tucson, A2: Institute for Mathematics and Education, University of Arizona).
- * Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

Solution Paths

- · What solution paths might you expect to see?
- · What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- · What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- · What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- · How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- · What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

On this page, you can write your thoughts on the following questions.

