1:5 Tyler's Grapes

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are

1:5 Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: ______________

Answer: Tyler has _____ grapes.

Answer

? - 8 = 6, 8 + 6 = ?, or another equivalent equation. Tyler has 14 grapes.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

Additional notes on the design of the task

The first sentence is a comparison statement between two quantities. The second sentence is simpler in that it refers to a single quantity. So if it helps students access the task, students could focus first on the second sentence.

initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 1:5 is called "Compare with Bigger Quantity Unknown."[‡] It is a Compare situation because subtraction is being used to compare the number of grapes Tyler has to the number of grapes Zoey has. More specifically, the situation is "Compare with Bigger Quantity Unknown," because the initially unknown quantity is the number of grapes Tyler has (and Tyler has more grapes than Zoey).

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

The *Progression* document has extensive discussion of aspects of teaching and learning about Compare problems, including the following points (pp. 12, 13).

- "One reason Compare problems are more advanced than the other two major types is that in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the "extra" that when added to the smaller unknown makes the total equal to the bigger unknown or by finding this quantity embedded within the bigger unknown."
- "The language of comparisons is also difficult. For example, 'Julie has three more apples than Lucy' tells both that Julie has more apples and that the difference is three. Many students 'hear' the part of the sentence about who has more, but do not initially hear the part about how many more; they need experience hearing and saying a separate sentence for each of the two parts in order to comprehend and say the one-sentence form."
- "Another language issue is that the comparing sentence might be stated in either of two related ways, using *more or less*. Students need considerable experience with *less* to differentiate it from *more*; some children think that *less* means *more*."

Task 1:5 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("14 grapes"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:5? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:5? In what specific ways do they differ from 1:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between addition and subtraction.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document (p. 13) stresses that "If textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Some equation models describe a situation in an algebraic way, such as ? - 8 = 6 (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as 8 + 6 = ? (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two one-digit numbers with sum greater than 10; writing equations to describe quantitative relationships; and fundamental concepts of addition and subtraction.

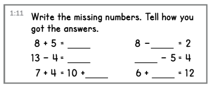
Extending the task

How might students drive the conversation further?

- For each number in an equation model for the situation, students could identify the quantity in the situation to which the number refers. For example, 8 refers to "the number of grapes Zoey has." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could ask or answer additional questions about the situation, such as "How many grapes do Tyler and Zoey have together?" and "If Tyler and Zoey share their grapes equally, how many grapes will each person have?"

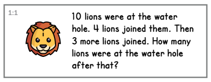
Related Math Milestones tasks

1:11

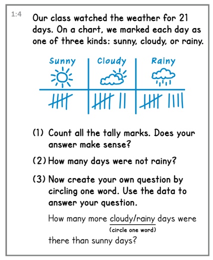

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ____ 8 − ____ = 2
13 − 4 = ____ ____ − 5 = 4
7 + 4 = 10 + ____ 6 + ____ = 12

1:1


1:1 10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

1:4


1:4 Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

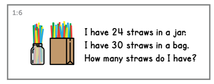
(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

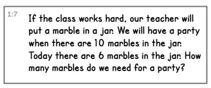
(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy (circle one word) days were there than sunny days?

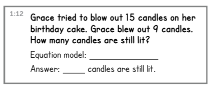
1:6


1:6 I have 24 straws in a jar. I have 30 straws in a bag. How many straws do I have?

1:7


1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

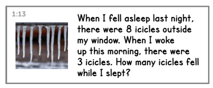
1:12


1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ____________

Answer: ____ candles are still lit.

1:13


1:13 When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Another task that involves adding two one-digit numbers with sum greater than 10 is the non-contextual task **1:11 Using Properties and Relationships**. Besides task 1:5, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) and *Compare with Difference Unknown ('how many more' language)* (part (3)); **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; **1:12 Blowing Out Candles**, *Put Together/Take Apart with One Addend Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown*.

In earlier and later grades, see the [Map of Addition and Subtraction Situations in K–2 Math Milestones](#).

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?