1:1 Lions at the Watering Hole

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

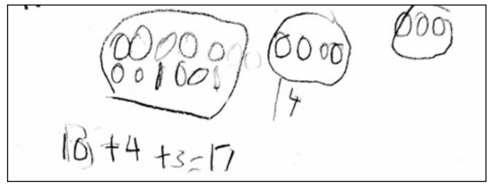
Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word

1:1

10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

Answer

There were 17 lions at the water hole after that. While the task does not require students to write an equation or represent the problem in other ways, a sample of student work is shown here.



Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.2, 1.OA; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

Multi-step problems like task 1:1 combine situation types. The action in task 1:1 is a sequence of two processes of "Add To with Result Unknown."[‡] In the first of these two processes, there are some lions at the water hole to begin with, and then more lions come, resulting in a new (initially unknown) group size. Then in the second process, still more lions arrive, resulting in a final (initially unknown) group size.

One approach to the problem is to identify the intermediate group size as a quantity worth knowing, and to calculate that quantity as 10 + 4 = 14. At that point the final group size can be calculated in another step as 14 + 3 = 17. An alternative strategy could be to combine the two "waves" of lion arrivals, 4 + 3 = 7, and then calculate the final group size as 10 + 7 = 17. The fact that the two strategies both work and both lead to the same result could be thought of as reflecting the basic "physics" of the situation in the task, and mathematically the agreement is guaranteed by the associative property of addition:

$$(10 + 4) + 3 = 10 + (4 + 3).$$

Grade 1 students might perform the calculations 10 + 4 and 10 + 7 simply by understanding the structure of the teen numbers (CCSS K.NBT.A.1 and 1.NBT.B.2b). A calculation of 14 + 3 might proceed by Level 2 or Level 3 methods; see the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20," pp. 14–17 in the *Progression* document. An example of a Level 2 method would be counting on from 14; thus, for 14 + 3, a student may say,

"Fourrrrteeeen, fifteen, sixteen, seventeen."

1 2 3

Students keep track of how many they counted on (here, 3) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Fourrrrteeeen…") is natural and indicates that the student differentiates between the first addend and the counts for the second addend.

Meanwhile, an example of a Level 3 method would be to think that 14 = 10 + 4, so that 14 + 3 = 10 + 4 + 3 = 10 + (4 + 3) = 10 + 7 = 17.

Students for whom the calculations in the task are time-consuming and/or effortful may need to be redirected to the context after obtaining the result 17, so as to relate the numbers in this equation to the context and answer the question in the task.

Additional notes on the design of the task

- The situation types being combined in the task are simple and familiar, to balance the fact that the situations are being combined in a multi-step problem.
- The starting number of 10 creates opportunities to learn and use the meaning of teen numbers.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:1? In what specific ways do they differ from 1:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two one-digit numbers with sum less than 10; writing numbers, expressions, and equations; working with teen numbers; and fundamental concepts of addition.

Extending the task

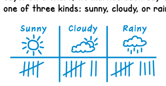
How might students drive the conversation further?

- Students could ask and answer additional questions about the situation. For example,
  - What if the 10 lions that were at the water hole first got scared away—how many lions would be at the water hole after they left?
  - A photographer took a picture that showed 11 of the lions. How many lions were not shown in the picture?
- Students could relate expressions or equations to the situation. For example, in the expression 10 + 4 + 3 what quantity in the situation does the number 10 refer to? (The number 10 refers to "the number of lions that were there first." Note that naming a quantity is different from naming the numerical value of the quantity.)

Related Math Milestones tasks

1:4

1:4 Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.



(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy (circle one word) days were there than sunny days?

1:5

1:5 Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: ________

Answer: Tyler has _____ grapes.

1:6

1:6

I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

1:7

1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

1:12

1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ________

Answer: _____ candles are still lit.

1:13

1:13

When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Besides task 1:1, other word problems and their situation types in grade 1 are as follows: tasks **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) and *Compare with Difference Unknown ('how many more' language)* (part (3)); **1:5 Tyler's Grapes**, *Compare with Bigger Quantity Unknown ('more' language)*; **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; **1:12 Blowing Out Candles**, *Put Together/Take Apart with One Addend Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown.*

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:2 Tens and Ones

Teacher Notes

Central math concepts

Building on their experiences in kindergarten with exploring teen numbers (see Teacher Notes for task **K:12 Make Ten and Some More**), students in grade 1 learn to see a collection of ten ones as a unit—called a ten—and they learn that the two digits of a two-digit number represent amounts of tens and ones. A special case of this idea is that the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. Another special case of this idea is that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

The *Progression* document[†] includes many useful points relevant to the teaching and learning of place value in grade 1, including the following:

- Although students were reciting the counting sequence to 100 by ones and by tens in kindergarten (CCSS K.CC.A.1), "The number words continue to require attention at first grade because of their irregularities. The decade words, 'twenty,' 'thirty,' 'forty,' etc., must be understood as indicating 2 tens, 3 tens, 4 tens, etc. Many decade number words sound much like teen number words. For example, 'fourteen' and 'forty' sound very similar, as do 'fifteen' and 'fifty,' and so on to 'nineteen' and 'ninety.' … [T]he number words from 13 to 19 give the number of ones before the number of tens. From 20 to 100, the number words switch to agreement with written numerals by giving the number of tens first. Because the decade words do not clearly indicate they mean a number of tens ('-ty' does mean tens but not clearly so) and because the number words 'eleven' and 'twelve' do not cue students that they mean '1 ten and 1' and '1 ten and 2,' children frequently make count errors such as 'twenty-nine, twenty-ten, twenty-eleven, twenty-twelve.'" (p. 6)
- Saying a two-digit number, 67 for example, as '6 tens, 7 ones' as well as 'sixty-seven' "can help students focus on the tens and ones structure of written numerals." (p. 6)
- "Comparing magnitudes of two-digit numbers uses the understanding that 1 ten is greater than any amount of ones represented by a one-digit number." (p. 8)
- When writing statements of comparison, "Correctly placing the < and > and symbols is a challenge for early learners. Accuracy can improve if students think of putting the wide part of the symbol next to the larger number." (p. 8)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: fluency with the count sequence within 100.

1:2

True or false?

6 tens + 4 ones < 4 ones + 7 tens

7 ones + 5 tens = _____

Answer

True. 57.

Click here for a student-facing version of the task.

Refer to the Standards

1.NBT.B; MP.1, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts

Additional notes on the design of the task

- The task is designed to target conceptual understanding, even though it only asks for brief answers rather than extended writing or other language demands. Teachers can also question students about the thinking that led to their answers, individually or in a group setting (and students can question each other).
- Place value units of tens and ones appear in both orders, so that relating the named quantities to base-ten numerals in positional notation is part of the task.

Extending the task

How might students drive the conversation further?

- Students could listen to each other's rationale for how they answered each question, restating it and checking that they restated it accurately.

- Students could create their own versions of task 1:2, trading their problems with a partner and checking each other's answers.

Related Math Milestones tasks

1:8

1:8
90 − 40 = ______
9 apples − 4 apples = ______ (number) ______ (unit)
9 cups − 4 cups = ______ (number) ______ (unit)
9 tens − 4 tens = ______ (number) ______ (unit)

1:10

1:10
Write the sum.

$$\begin{array}{r} 37 \\ +\ 46 \\ \hline \end{array}$$

1:6

1:6

I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

Tasks **1:8 Subtracting Units** and **1:10 Two-Digit Addition** involve calculating with two-digit numbers based on place value understanding. Task **1:6 Two Groups of Straws** involves a sum of two two-digit numbers in context.

2:2

2:2
(1) True or false?
(a) 2 hundreds + 3 ones > 5 tens + 9 ones
(b) 9 tens + 2 hundreds + 4 ones < 924
(c) 456 < 5 hundreds

(2) Write the number that makes each statement true.
(a) 7 ones + 5 hundreds = ____
(b) 14 tens = ____
(c) 90 + 300 + 4 = ____

5:4

5:4
(1) Circle T for true or F for false.
(a) 9 thousandths + 5 hundredths > 3 hundredths + 2 tenths T F
(b) 92 hundredths + 4 thousandths > 0.924 T F
(c) 0.456 < 0.5 T F

(2) Write each number in the requested form.
(a) 7 thousandths + 5 tenths = ____ (decimal)
(b) 0.1 tenths = ____ (decimal)
(c) $\frac{2}{100} + \frac{5}{1000}$ = ____ (decimal)
= ____ (fraction in lowest terms)

8:6

8:6
Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$.
(2) $3^2 \cdot 3^{-5}$.

In later grades, task **2:2 Place Value to Hundreds** continues the progression of place value concepts; these concepts extend to finite decimals in task **5:4 Place Value to Thousandths** and to repeating decimals in task **8:6 Rational Form**.

K:12

K:12
Draw 16 circles. Use a [favorite color] marker for 10 of them. Use a pencil for the rest. [Student draws.]
How many are [favorite color]? How many are in pencil?
Write the missing number: 16 = 10 + ____

In earlier grades, task **K:12 Make Ten and Some More** involves understanding the number 16 as ten and six more.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:2? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:2? In what specific ways do they differ from 1:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Number and Operations in Base Ten.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression* document.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:3 Paper Clip Length Units

Teacher Notes

Central math concepts

Building on their experiences with comparing measurable attributes directly in kindergarten (CCSS K.MD.A), students in grade 1 measure lengths using an object as a length unit. Far from being a procedural task, length measurement in this grade is an imaginative act that involves "choosing a unit of measure and *subdividing* (mentally and physically) the object by that unit, placing that unit end to end (*iterating*) alongside the object" (*Progression* document,[†] p. 4). The length of an object is thus the number of units required to iterate from one end of the object to the other, without gaps or overlaps. This last phrase, "without gaps or overlaps," will recur throughout geometric measurement—for area in grade 3, for angle in grade 4, and for volume in grade 5 (see, respectively, the Teacher Notes for tasks **4:13 Area Units**, **4:8 Shapes with Given Positions**, and **5:3 Neighborhood Garden**).

As noted in the *Progression* document (p. 3), "The concept of unit is crucial."[‡] The concept of a unit underlies not only geometric measurement, but also measurement of every quantity, whether that be mass, time, or a derived quantity such as speed, population density, person-hours, and so on. Additionally, the concept of a unit is essential to thinking about fractions and about place value. For example, grade 1 students begin the study of place value by working with units of tens and ones (see the Teacher Notes for tasks **1:2 Tens and Ones** and **1:8 Subtracting Units**).

Returning to the topic of task 1:3, "Length is a core concept for several reasons. It is the basic geometric measurement. It is also involved in area and volume measurement, especially once formulas are used. Length and unit iteration are critical in understanding and using the number line.… Length is also one of the most prevalent metaphors for quantity and number… the master metaphor for magnitude" (p. 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: spatially structuring a line into sections.

Extending the task

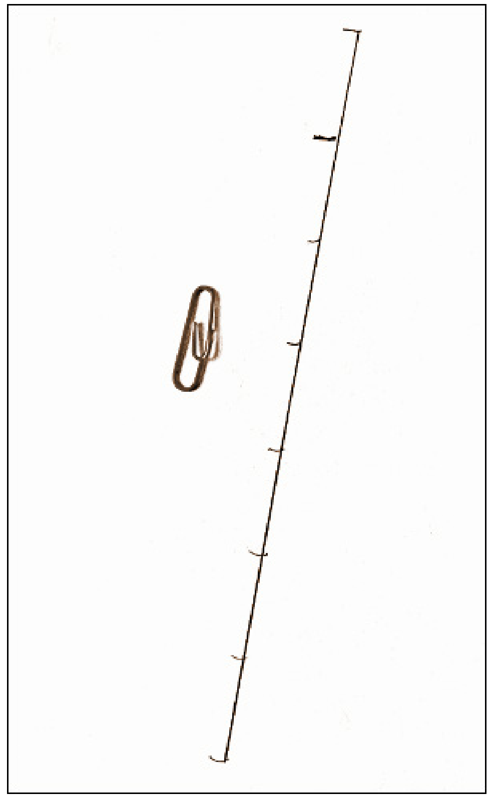
How might students drive the conversation further?

- Students could use their drawing to cut out a rectangle 7 paper-clip-units long (shaped like a ruler). They could use the rectangle as a measurement scale to measure objects in the classroom in units of paper-clip-lengths.
- If the students put all their cut-out rectangles together on one desk, are they all pretty close to the same length? What is the value of having all of the rulers at the school-supply store marked the same way?

1:3 Using a paper clip as a unit of length, draw a straight line 7 units long.

Answer

See example picture.



Click here for a student-facing version of the task.

Refer to the Standards

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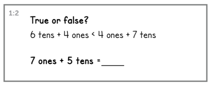
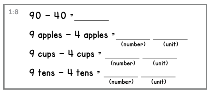
Related Math Milestones tasks

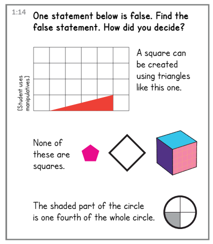
1:2



1:8

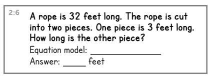


1:14

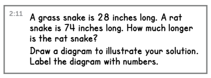


Tasks **1:2 Tens and Ones** and **1:8 Subtracting Units** involve place value units. Task **1:14 Shape True/False** involves squares, which have a defining attribute based on length (all four sides the same length).

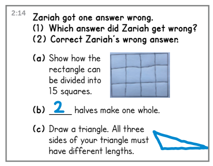
2:6



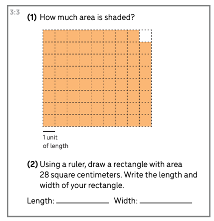
2:11



2:14



3:3



In later grades, tasks **2:6 Cutting a Rope** and **2:11 Grass Snake vs. Rat Snake** both involve addition, subtraction, and length in context. Task **2:14 Correcting a Shape Answer** involves side lengths in a triangle as well as spatial structuring. Task **3:3 Length and Area Quantities** relates and distinguishes between these two quantities and their units.

K:14

In earlier grades, task **K:14 Animals from Land and Sea** involves a choice of unit (either "animals" or a more specific choice of "land animals"/"sea animals"), which equates to a choice of what to count.

Aspect(s) of rigor

Concepts

Additional notes on the design of the task

The task involves producing a figure with given length; this lays greater emphasis on the *concept* of measurement and less emphasis on the *procedure* of measurement (as compared with a task that only asks students to measure a given object).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:3? In what specific ways do they differ from 1:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2012, June 23). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometric Measurement*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression* document.

‡ See also Zimba (2013), "Units, a Unifying Theme in Measurement, Fractions, and Base Ten."

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
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Identity, Agency, and Belonging

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- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:4 Analyzing Weather Data

Teacher Notes

Central math concepts

Students' data work in the primary grades involves representing and interpreting categorical data, which is data that arises from classifying or sorting into categories.† In a chart like the one shown in task 1:4, the tally marks are the individual data points. Interpreting such a chart involves grasping the correspondence between the tally marks and the facts they represent. It is a significant act of the imagination to look at a tally mark in the first column and see in that mark a historical record of a recent sunny day.

From the individual data points (the tally marks), students can use counting to determine the number of data points in each category (5, 7, and 9) and the total count, 21. These counts are numerical summaries of the data. The counts can then be used to analyze the data—to ask and answer questions about the data.

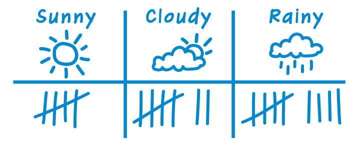
In this way, students' analysis of categorical data connects directly to their uses of addition and subtraction to solve problems in context. In grade 1 specifically, when students "ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another" (CCSS 1.MD.C.4), they are using "addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions" (CCSS 1.OA.A.1). More generally, there are close connections in every elementary grade between students' data work and their expanding use of numbers and operations in context; see Table 1, p. 4 of the *Progression* document for a list of these connections in grades K–5.

As the *Guidelines for Assessment and Instruction in Statistics Education Report* notes, "data are not just numbers, they are numbers with a context. In mathematics, context obscures structure. In data analysis, context provides meaning."‡ Thus as the *Progression* document notes, "students should work with data in the context of science, social science, health, and other subjects, always interpreting data plots in terms of the context they represent" (p. 3).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: cardinal counting; skip-counting; comparing numbers; adding and subtracting single-digit numbers; subitizing groups of 5; and using addition and subtraction to solve problems in context.

1:4 Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.



(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy days were (circle one word) there than sunny days?

Answer

(1) There are 21 tally marks. That makes sense because the class made one tally mark every day for 21 days. **(2)** 12 days were not rainy. **(3)** Possible questions/answers: "How many more cloudy days were there than sunny days?"—There were 2 more cloudy days than sunny days. "How many more rainy days were there than sunny days?"—There were 4 more rainy days than sunny days.

Click here for a student-facing version of the task.

Refer to the Standards

1.MD.C.4; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Application

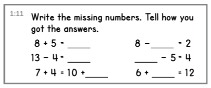
Extending the task

How might students drive the conversation further?

- Students could ask additional questions about the data, such as which kind of day was most common and which kind of day was least common.
- Students could ask questions similar to those in the task to analyze another set of data they have collected in the classroom.

Related Math Milestones tasks

1:11


1:11 Write the missing numbers. Tell how you got the answers.

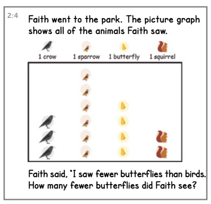
8 + 5 = ____ 8 − ____ = 2

13 − 4 = ____ ____ − 5 = 4

7 + 4 = 10 + ____ 6 + ____ = 12

Task **1:11 Using Properties and Relationships** involves sums of single-digit numbers that cross ten, as task 1:4 does.

2:4



Task **2:4 Animals in the Park** combines situation types "Put Together with Total Unknown" and "Compare with Difference Unknown" in a context involving a picture graph display of categorical data.

K:14

In earlier grades, task **K:14 Animals from Land and Sea** involves a comparison based on classifying into categories.

See the Map of Addition and Subtraction Situations in K–2 Math Milestones to find tasks in grades K, 1, and 2 that cover the various addition and subtraction situation types.

Additional notes on the design of the task

- The situation types involved in the task are "Put Together with Total Unknown" and "Compare with Difference Unknown."
- The tally marks can be used to relate the analysis of the data to place value and properties of operations. For example, suppose we ask how many days weren't sunny. This number is visible on the chart as two groups of 5, a group of 2, and a group of 4. The total, 16, can be arrived at by adding (5 + 2) + (5 + 4), which corresponds to the sum 7 + 9, or it could be arrived at by (5 + 5) + (2 + 4), which corresponds to the sum 10 + 6. The equivalence of (5 + 2) + (5 + 4) and (5 + 5) + (2 + 4) illustrates the 'any which way' principle (a combination of the commutative and associative properties of addition). And the equation 7 + 9 = 10 + 6 records the 'making ten' process for 7 + 9.
- The option to circle words allows the task to involve student-created questions. Alternatively, the possible combinations/questions could be generated in advance, and students could choose which ones to solve.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:4? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:4? In what specific ways do they differ from 1:4?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, June 20). *Progressions for the Common Core State Standards in Mathematics (draft): K–3, Categorical Data; Grades 2–5, Measurement Data.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ The Guidelines for Assessment and Instruction in Statistics Education Report was published in 2007 by the American Statistical Association, http://www.amstat.org/education/gaise.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:5 Tyler's Grapes

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are

1:5 Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: ____________

Answer: Tyler has _____ grapes.

Answer

? - 8 = 6, 8 + 6 = ?, or another equivalent equation. Tyler has 14 grapes.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

Additional notes on the design of the task

The first sentence is a comparison statement between two quantities. The second sentence is simpler in that it refers to a single quantity. So if it helps students access the task, students could focus first on the second sentence.

initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 1:5 is called "Compare with Bigger Quantity Unknown."[‡] It is a Compare situation because subtraction is being used to compare the number of grapes Tyler has to the number of grapes Zoey has. More specifically, the situation is "Compare with Bigger Quantity Unknown," because the initially unknown quantity is the number of grapes Tyler has (and Tyler has more grapes than Zoey).

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

The *Progression* document has extensive discussion of aspects of teaching and learning about Compare problems, including the following points (pp. 12, 13).

- "One reason Compare problems are more advanced than the other two major types is that in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the "extra" that when added to the smaller unknown makes the total equal to the bigger unknown or by finding this quantity embedded within the bigger unknown."
- "The language of comparisons is also difficult. For example, 'Julie has three more apples than Lucy' tells both that Julie has more apples and that the difference is three. Many students 'hear' the part of the sentence about who has more, but do not initially hear the part about how many more; they need experience hearing and saying a separate sentence for each of the two parts in order to comprehend and say the one-sentence form."
- "Another language issue is that the comparing sentence might be stated in either of two related ways, using *more or less*. Students need considerable experience with *less* to differentiate it from *more*; some children think that *less* means *more*."

Task 1:5 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("14 grapes"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:5? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:5? In what specific ways do they differ from 1:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between addition and subtraction.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document (p. 13) stresses that "If textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Some equation models describe a situation in an algebraic way, such as ? - 8 = 6 (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as 8 + 6 = ? (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two one-digit numbers with sum greater than 10; writing equations to describe quantitative relationships; and fundamental concepts of addition and subtraction.

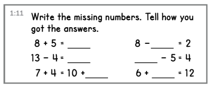
Extending the task

How might students drive the conversation further?

- For each number in an equation model for the situation, students could identify the quantity in the situation to which the number refers. For example, 8 refers to "the number of grapes Zoey has." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could ask or answer additional questions about the situation, such as "How many grapes do Tyler and Zoey have together?" and "If Tyler and Zoey share their grapes equally, how many grapes will each person have?"

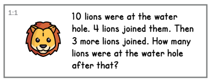
Related Math Milestones tasks

1:11

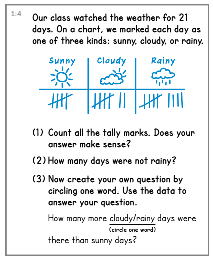

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ____ 8 − ____ = 2
13 − 4 = ____ ____ − 5 = 4
7 + 4 = 10 + ____ 6 + ____ = 12

1:1


1:1 10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

1:4


1:4 Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

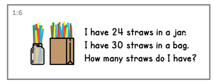
(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

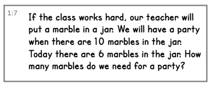
(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy (circle one word) days were there than sunny days?

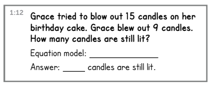
1:6


1:6 I have 24 straws in a jar. I have 30 straws in a bag. How many straws do I have?

1:7


1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

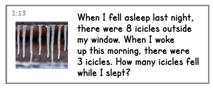
1:12


1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ______________

Answer: _____ candles are still lit.

1:13


1:13 When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Another task that involves adding two one-digit numbers with sum greater than 10 is the non-contextual task **1:11 Using Properties and Relationships**. Besides task 1:5, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) and *Compare with Difference Unknown ('how many more' language)* (part (3)); **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; **1:12 Blowing Out Candles**, *Put Together/Take Apart with One Addend Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown*.

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students’ interests, experiences, or funds of knowledge?
- How can you build students’ self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students’ agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students’ self-confidence?

1:6 Two Groups of Straws

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

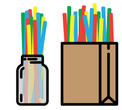
1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

1:6



I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

Answer

I have 54 straws.

Click here for a student-facing version of the task.

Refer to the Standards

1.NBT.C, 1.OA.A; MP.1, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Procedural skill and fluency, Application

Additional notes on the design of the task

- The task balances the simplicity and familiarity of the situation type with the complexity and newness of the required calculation. (The situation type is more simple and familiar, which is balanced by the fact that the required calculation is more complex and new for students.)
- Although 54 straws is a large number, it isn't unreasonable to deal with such a quantity if, for example, the straws will be used as supplies for a craft project.

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 1:6 is called "Put Together/Take Apart with Both Addends Unknown."[‡] It is a Put Together/Take Apart situation because two groups of straws are being put together—that is, two separate groups of straws are being imagined as a single group. More specifically, the situation is "Put Together/Take Apart with Total Unknown," because the initially unknown quantity is the total number of straws.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: working with the count sequence within 100; and adding two two-digit numbers by adding tens and tens, ones and ones.

Extending the task

How might students drive the conversation further?

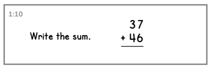
- Students could ask and answer additional questions about the situation. For example,
  - How many more straws are in the bag than in the jar?
  - If I use 10 of the straws for an art project, how many straws will I have left?
- Students could relate expressions and equations to the situation. For example, suppose I move 4 straws from the jar to the bag. How many straws will be in the jar and how many straws will be in the bag? How many straws will there be altogether? Students could discuss these problems in relation to the equation 24 + 30 = 20 + 34.

Curriculum connection

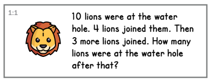
1. In which unit of your curriculum would you expect to find tasks like 1:6? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:6? In what specific ways do they differ from 1:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

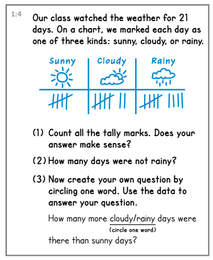
1:10



1:1



1:4



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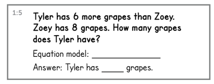
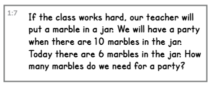
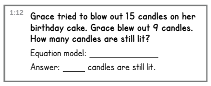


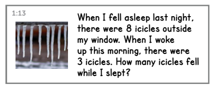
1:7



1:12



1:13



Another task that involves adding two two-digit numbers is the non-contextual task **1:10 Two-Digit Addition**. Besides task 1:6, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) and *Compare with Difference Unknown ('how many more' language)* (part (3)); **1:5 Tyler's Grapes**, *Compare with Bigger Quantity Unknown ('more' language)*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; **1:12 Blowing Out Candles**, *Put Together/Take Apart with One Addend Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown.*

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:7 Class Marble Jar

Teacher Notes

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

Answer

We need 4 marbles for a party.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation in task 1:7 can be thought of as "Add To with Change Unknown."[‡] Some marbles must be added to the jar, but initially it is unknown how many marbles must be added.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding. In terms of the situation in task 1:7, the number of marbles that must be *added* can be found by *subtraction*: 10 - 6 equals 4 because 6 + 4 equals 10.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: subtracting two one-digit numbers; adding two one-digit numbers; and fundamental concepts of addition and subtraction.

Extending the task

How might students drive the conversation further?

- Students could ask and answer additional questions about the situation. For example, "What if the teacher decided that 15 marbles were needed for a party—how many marbles would be needed, if we have 6 marbles now?" Students could discuss and compare two different approaches such as 15 - 6 = ? and 4 + 5 = 9. (The first way repeats the analysis from the beginning; the second way says that since 5 more marbles are now needed for a party, the class needs 5 more marbles than it needed before.)

- Students could create their own version of the original task by choosing a different number of marbles to be in the jar now.

Additional notes on the design of the task

The calculation in task 1:7 involves partners of 10 (6 and 4). Working with partners of 10 begins in kindergarten (CCSS K.OA.A.4). Remembering partners of 10 is useful in grade 1 for carrying out make-a-ten methods for adding within 20, as in the problem 8 + 5 = 8 + 2 + 3 = 10 + 3 = 13.[§]

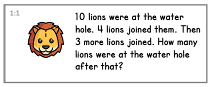
Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:7? In what specific ways do they differ from 1:7?

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

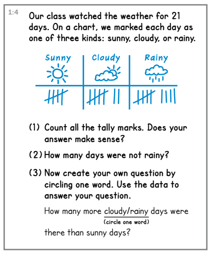
1:1



1:1

10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

1:4



1:4

Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

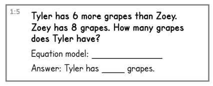
(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy days were there than sunny days?
(circle one word)

1:5



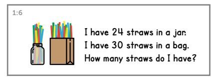
1:5

Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: ______________

Answer: Tyler has _____ grapes.

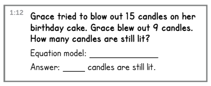
1:6



1:6

I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

1:12



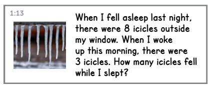
1:12

Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ______________

Answer: _____ candles are still lit.

1:13



1:13

When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Besides task 1:7, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) *and Compare with Difference Unknown ('how many more' language)* (part (3)); **1:5 Tyler's Grapes**, *Compare with Bigger Quantity Unknown ('more' language)*; **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:12 Blowing Out Candles**, *Put Together/Take Apart with One Addend Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown*.

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

§ Zimba, Jason. (2016). How I see addition facts. Colorado Mathematics Teacher, 49(3), Article 5. https://digscholarship.unco.edu/cmt/vol49/iss3/5/

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:8 Subtracting Units

Teacher Notes

Central math concepts

What can you count? Young students are familiar with counting *things*, like apples. But what about "tens"? Tens aren't physical objects, yet they can be counted. Who decides what can be counted and what can't? Students should have experience deciding what to count so they learn how counting extends from prominent objects like apples to abstract things like tens. Developing student agency in deciding what to count extends to deciding what units to count. A unit is what the number 1 refers to in a count. Here they count tens; in later grades they will count unit fractions as in "9 thirds - 4 thirds = 5 thirds."

As reflected in task **K:12 Make Ten and Some More**, when students were in kindergarten, they decomposed numbers 10–19 into ten ones and some more ones. That task only requires working explicitly with units of ones. In first grade however, students will begin their study of place value in earnest by working with a larger unit, called "a ten." That is, students learn to view ten ones as a unit called a ten.

One application of the tens unit is that it allows students to interpret the "decade numbers" 10, 20, 30, 40, 50, 60, 70, 80, 90 as referring to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). This understanding in turn allows students to add or subtract decade numbers by performing single-digit calculations with tens units. For example, the sum 50 + 30 is 5 tens + 3 tens, which makes 8 tens; and 8 tens are 80. So, 50 + 30 = 80.

A linguistic challenge of the place value system is that in the English language, it takes some interpretation to understand that "fifty" means five tens, "twenty" means two tens, and so on. By contrast, the English names for hundred words indicate the place value unit more directly; for example, the name "three hundred" says almost exactly what it means (three hundreds). Additional aspects of naming numbers in relation to the place value system are detailed in the relevant *Progression* document† (see pp. 5, 6).

At first, using unit thinking to calculate with decade numbers might not be the method best preferred by all students. For example, given the problem 50 + 30 = ?, some students might prefer to approach the problem by skip-counting by tens ("sixty, seventy, eighty") to obtain the result 80. That method is valid and efficient for the stated problem, but basing calculation explicitly on place-value units will become increasingly important as students progress through the elementary grades. Not only that, but operations with fractions and decimals depend heavily on unit thinking. So it is important to connect a method like skip-counting by tens to the method of adding or subtracting decade numbers by performing single-digit calculations with tens units.

1:8

90 − 40 = ______

9 apples − 4 apples = ______ (number) ______ (unit)

9 cups − 4 cups = ______ (number) ______ (unit)

9 tens − 4 tens = ______ (number) ______ (unit)

Answer

50. 5 apples. 5 cups. 5 tens.

Click here for a student-facing version of the task.

Refer to the Standards

1.NBT.C.6; MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts

Additional notes on the design of the task

Task 1:8 is designed to target conceptual understanding, even though it only asks for brief answers rather than asking for extended writing or making other language demands. Teachers can also question students about the thinking that led to their answers, individually or in a group setting (and students can question each other).

Helping students see how non-place-value-based methods connect to place-value- based methods can help students who use non-place-value-based methods transition to place-value based methods, and making those connections can also enable students who use place-value-based methods to better understand their own method. Conversations between and among students that foster such connections can be aided by manipulatives and diagrams.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: subtracting a one-digit number from a one-digit number; interpreting decade numbers as naming single-digit numbers of tens units; and fluency with the count sequence within 100.

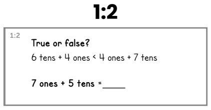
Extending the task

How might students drive the conversation further?

- Students could be asked about how the different parts of this task relate to each other (for example, the first part and the last part).
- Students could create their own versions of the task by beginning with a different problem involving subtraction (or addition) of decade numbers, and using different choices for the units in the second and third step. Students could give their version of the task to a partner and discuss each other's answers.

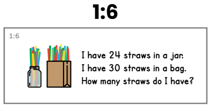
Related Math Milestones tasks

1:2

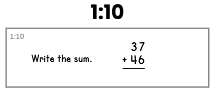

1:2
True or false?
6 tens + 4 ones < 4 ones + 7 tens

7 ones + 5 tens = _____

1:6


1:6
I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

1:10


1:10
Write the sum.
$$\begin{array}{r} 37 \\ +\ 46 \\ \hline \end{array}$$

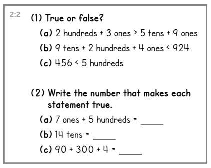
Task **1:2 Tens and Ones** is a conceptual task that involves connections between place value units and positional notation for two-digit numbers. Task **1:6 Two Groups of Straws** is a contextual problem whose solution involves the sum 24 + 30, which can be approached by adding tens and

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:8? In what specific ways do they differ from 1:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

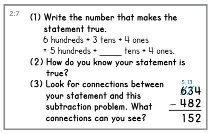
tens, ones and ones. Task **1:10 Two-Digit Addition** is a computation task that involves the sum 37 + 46, which can be approached by adding tens and tens, ones and ones, and composing a ten.

2:2



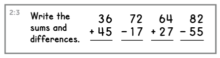
2:2 (1) True or false?
(a) 2 hundreds + 3 ones > 5 tens + 9 ones
(b) 9 tens + 2 hundreds + 4 ones < 924
(c) 456 < 5 hundreds
(2) Write the number that makes each statement true.
(a) 7 ones + 5 hundreds = ____
(b) 14 tens = ____
(c) 90 + 300 + 4 = ____

2:7



2:7 (1) Write the number that makes the statement true.
6 hundreds + 3 tens + 4 ones = 5 hundreds + ____ tens + 4 ones.
(2) How do you know your statement is true?
(3) Look for connections between your statement and this subtraction problem. What connections can you see?
634 − 482 = 152

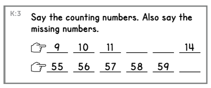
2:3



2:3 Write the sums and differences.
36 + 45, 72 − 17, 64 + 27, 82 − 55

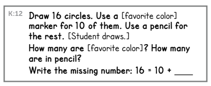
In later grades, task **2:2 Place Value to Hundreds** is a conceptual task that involves the extension of place value from tens to hundreds. Task **2:7 Subtraction Regrouping** connects place value concepts to calculation procedures for subtracting three-digit numbers. Task **2:3 Fluency within 100 (Add/Subtract)** is a fluency task involving calculation of two-digit sums and differences.

K:3



K:3 Say the counting numbers. Also say the missing numbers.
9 10 11 ___ ___ 14
55 56 57 58 59 ___

K:12



K:12 Draw 16 circles. Use a [favorite color] marker for 10 of them. Use a pencil for the rest. [Student draws.]
How many are [favorite color]? How many are in pencil?
Write the missing number: 16 = 10 + ___

In earlier grades, task **K:3 Say the Numbers (Teens, Decades)** involves the counting sequence for two-digit numbers. Task **K:12 Make Ten and Some More** involves the meaning of 16 as ten ones and 6 more ones.

† Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Number and Operations in Base Ten*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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Identity, Agency, and Belonging

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- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:9 Fluency within Ten

Teacher Notes

Central math concepts

During grade 1, repeated experiences with adding and subtracting will help students know the partner that makes 10 for any number and know all decompositions for any number less than 10. This work is a continuation of kindergarten problem solving (CCSS K.OA.A.4, K.OA.A.3). Knowing all the partners of 10 and the decompositions of all the numbers less than 10 supports extending addition and subtraction to larger problems within 20; see the Teacher Notes for task **1:11 Using Properties and Relationships** and the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20" in the relevant *Progression* document,[†] pp. 14–17.

Grade 1 students solve problems in addition and subtraction within 20. Building on the kindergarten fluencies (CCSS K.OA.A.5), the grade 1 fluency goal is addition and subtraction within 10: that is, fluently finding or remembering[‡] sums of two addends with total 10 or less, and fluently finding the related differences, sometimes by remembering the relevant decomposition (CCSS 1.OA.C.6). Task 1:9 is keyed to this fluency goal: the task involves two sums of two addends with total 10 or less (4 + 5 and 2 + 6), two differences related to sums of two addends with total 10 or less (10 - 8 and 7 - 4), and two cases of partners of 10 (the unknown addend problems 4 + ☐ = 10 and 7 + ☐ = 10).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting on; finding partners of 10 and partners of smaller numbers; and using the relationship between addition and subtraction.

Extending the task

How might students drive the conversation further?

- For each equation in task 1:9, students could write the rest of the equations in the fact family. For example, 7 - 4 = 3 ⇨ 7 - 3 = 4, 3 + 4 = 7, 4 + 3 = 7.

1:9 Write the missing numbers.

4 + 5 = ____	7 − 4 = ____
10 − 8 = ____	2 + 6 = ____
4 + ____ = 10	7 + ____ = 10

Answer

Left column: 9, 2, 6. *Right column*: 3, 8, 3.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.C.6; MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Procedural skill and fluency

Additional notes on the design of the task

The task does not include a direction to "Tell how you got the answers" because the focus of the task is procedural skill and fluency.

Related Math Milestones tasks

1:7

1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

1:12

1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ____________

Answer: _____ candles are still lit.

1:11

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = _____	8 − _____ = 2
13 − 4 = _____	_____ − 5 = 4
7 + 4 = 10 + _____	6 + _____ = 12

Task **1:7 Class Marble Jar** involves finding a partner of 10 in context. Solving task **1:12 Blowing Out Candles** involves finding the difference 8 - 3. Task **1:11 Using Properties and Relationships** includes sums and differences involving totals greater than 10, as well as equations of the form *C* - ☐ = *B* and ☐ - *A* = *B*.

2:5

2:5 Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2 4✓

Click here for student handout 2:5

2:8

2:8 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 2:8

In later grades, tasks **2:5 Sums of Single-Digit Numbers** and **2:8 Fluency within the Addition Table** represent the culminating fluency and remembering goals for addition and subtraction within 20 (CCSS 2.OA.B.2).

K:5

K:5 [Teacher puts 3 red counters on table.] **Put some blue counters here to make 10 counters in all.** [Student completes this task.] **How many counters did you add?** [Student determines the answer.]

Write the missing number: 3 + ____ = 10

K:8

K:8 [Teacher holds out 5 paper clips.] **How many do I have?** [Student counts the paper clips.] [Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?** [Student counts the paper clips.]

How many are in my other hand?

K:13

K:13 Write or say the missing numbers.

3 + 1 = _____	2 + 3 = _____
5 + 0 = _____	2 − 2 = _____
4 − 3 = _____	5 − 3 = _____

In earlier grades, task **K:5 Adding to Make a Group of Ten** involves finding a partner of 10, and task **K:8 Five Behind the Back** involves decompositions of 5. Task **K:13 Fluency within Five** concentrates on problems within the kindergarten fluency goal (CCSS K.OA.A.5).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:9? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:9? In what specific ways do they differ from 1:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ Although some or many grade 1 students will gain enough experiences during the year to know a few, or more than a few, single-digit sums from memory by the end of the year, remembering all the single-digit sums isn't a grade 1 expectation (CCSS 2.OA.B.2)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

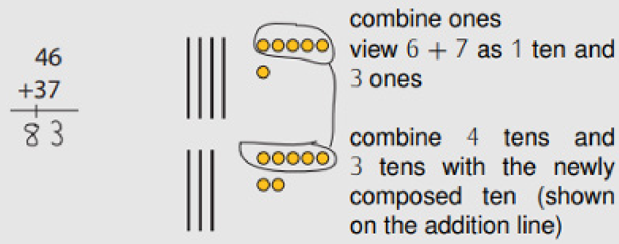
1:10 Two-Digit Addition

Teacher Notes

Central math concepts

Having learned that the two digits of a two-digit number represent amounts of tens and ones (see Teacher Notes for task **1:2 Tens and Ones**), students are in a position to calculate sums of two two-digit numbers with total 100 or less, by adding tens and tens, ones and ones, and composing a ten as needed.

Adding tens and ones separately



This method is an application of the commutative and associative properties. The diagrams can help children with understanding and explaining the steps (MP.1). Advantages of writing the 1 below the addends are discussed in the Grade 2 margin.

Adding ones and ones in the problem 37 + 46 will confront students with the subproblem 7 + 6 = ?, which is itself a nontrivial problem at this grade (CCSS 1.OA.C.6). Students can add ones and ones using so-called Level 2 or Level 3 methods (see the *Progression* document,† pp. 14–17 and the Teacher Notes for task **1:11 Using Properties and Relationships**), and/or in some cases by knowing the value of the sum from memory. When the sum of the ones digits is 10 or more, students compose a ten, as shown in the figure. The figure is from the NBT *Progression* document‡ (pp. 6, 7), which notes that "[c]oncrete objects, cards, or drawings afford connections with written numerical work and discussions and explanations in terms of tens and ones. In particular, showing composition of a ten with objects or drawings affords connection of the visual ten with the written numeral 1 that indicates 1 ten." (Note that the objects, cards, or drawings are not methods, but serve to explain and allow discussion of the numerical work.)

There are other ways to add two two-digit numbers using concepts of place value and properties of addition, such as by counting on by tens (46 + 37 = "56, 66, 76…77-78-79-80-81- 82- 83") or by using an opportunistic strategy such as 46 + 37 = 46 + 4 + 33 = 50 + 33 = 83. But using the concept of place value units to add tens and tens, ones and ones is thinking that generalizes directly and efficiently to more complex problems that students will encounter in grade 2 (see Teacher Notes for task **2:10 Three Digit Addition/Subtraction**).

1:10

Write the sum.

$$\begin{array}{r} 37 \\ +\ 46 \\ \hline \end{array}$$

Answer

83.

Click here for a student-facing version of the task.

Refer to the Standards

1.NBT.C.4; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Procedural skill and fluency

Additional notes on the design of the task

In the sum 37 + 46, both addends have some tens and some ones, and it is necessary to compose a ten.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:10? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 1:10? In what specific ways do they differ from 1:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: concepts of tens and ones; and adding two two-digit numbers.

Extending the task

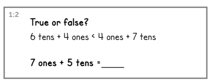
How might students drive the conversation further?

- Students could discuss whether they prefer to add the tens first or add the ones first. (See the relevant *Progression* document, p. 7.)
- Students could consider a three-addend problem with total less than 100, and with no composing a ten required, such as the one shown.

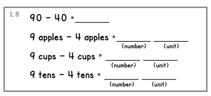
$$\begin{array}{r} 32 \\ 13 \\ +24 \\ \hline \end{array}$$

Related Math Milestones tasks

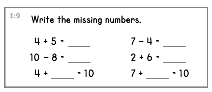
1:2



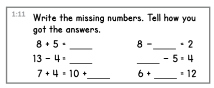
1:8



1:9

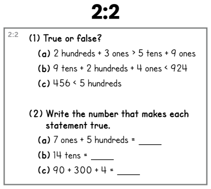


1:11

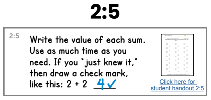


Place value ideas within 100 are the subject of task **1:2 Tens and Ones**. Task **1:8 Subtracting Units** portrays a two-digit subtraction problem as a matter of subtracting two single-digit numbers of tens units. Sums of single-digit numbers are involved in tasks **1:9 Fluency within Ten** and **1:11 Using Properties and Relationships**.

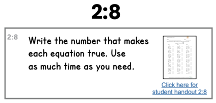
2:2



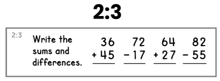
2:5



2:8

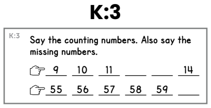


2:3

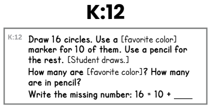


Place value concepts are the subject of task **2:2 Place Value to Hundreds**. Tasks **2:5 Sums of Single-Digit Numbers** and **2:8 Fluency within the Addition Table** involve the single-digit sums and related differences upon which multi-digit addition and subtraction algorithms are built. Two-digit sums and differences are addressed from a fluency perspective in task **2:3 Fluency within 100 (Add/Subtract)**.

K:3



K:12



In earlier grades, task **K:3 Say the Numbers (Teens, Decades)** involves the count sequence for two-digit numbers, and task **K:12 Make Ten and Some More** involves the structure of a teen number as ten ones and some more ones.

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in this Teaching Note refer to this *Progression* document.

‡ Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Number and Operations in Base Ten*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

Central math concepts

A major development in adding and subtracting during grade 1 is the progression from Level 1 methods that directly model the quantities and operations involved to Level 2 methods that involve counting-on and then to Level 3 methods that use properties of operations and relationships between operations to replace a given problem with an easier problem. See the figure, from the *Progression* document,[†] p. 6, and the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20," pp. 14–17.

Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away. Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2. Counting On. Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3. Convert to an Easier Problem. Decompose an addend and compose a part with another addend.

An important example of a Level 3 strategy is the make-a-ten method for single-digit addends with sum greater than 10. For example, to use the make-ten strategy to calculate the sum 8 + 5 in task 1:11, students can think of the sequence of calculations

$$8 + 5 = 8 + 2 + 3 = 10 + 3 = 13.$$

The strategy requires providing the number (2) that makes 10 when added to 8, and providing the number (3) that makes 5 when added to 2. In the last step, the strategy leverages understanding of the meaning of teen numbers. The *Progression* document (p. 16) summarizes these "three prerequisites that reach back into kindergarten":[‡]

a) knowing the partner that makes 10 for any number (CCSS K.OA.A.4 sets the stage for this),

b) knowing all decompositions for any number below 10 (CCSS K.OA.A.3 sets the stage for this), and

c) knowing all teen numbers as 10 + n (e.g., 12 = 10 + 2, 15 = 10 + 5, see CCSS K.NBT.A.1 and 1.NBT.B.2b).

The problem 7 + 4 = 10 + ? in task 1:11 is a sort of snapshot of the make-a-ten method in progress. The equation 7 + 4 = 10 + 1 can be interpreted as a result of decomposing 4 = 3 + 1.

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ____	8 − ____ = 2
13 − 4 = ____	____ − 5 = 4
7 + 4 = 10 + ____	6 + ____ = 12

Answer

Left column: 13, 9, 1. *Right column*: 6, 9, 6. Student explanations may vary (see "Central math concepts").

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.B; MP.1, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Procedural skill and fluency

Make-a-ten methods are Level 3 because they use properties of addition to replace a given problem with an easier problem. Make-a-ten methods also take advantage of place value, because of the role of 10 in the strategy.

An example of a Level 2 method would be finding the difference 13 - 9 by counting on. Paraphrasing the *Progression* document (p. 14), this approach involves seeing the 9 as part of 13, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9: thus the student may say,

"Niiiiine, ten, eleven, twelve, thirteen."
1 2 3 4

Students keep track of how many they counted on (here, 4) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Niiiiine...") is natural and indicates that the student differentiates between the first addend and the counts for the second addend. Counting up from 9 to subtract is preferable than counting down from 13 to subtract, both because counting up is easier and also because counting up reinforces the relationship between addition and subtraction.

Compared to 13 - 9, the problem 13 - 4 in task 1:11 is perhaps harder to solve using Level 2 methods. A possible Level 3 method for 13 - 4 would be to think that

$$\begin{aligned} 13 - 4 &= 13 - 3 - 1 \\ &= 10 - 1 \\ &= 9. \end{aligned}$$

Alternatively, a different Level 3 method would be to think of 13 - 4 as an unknown addend problem, 4 + ☐ = 13, and then think that since 4 + 6 = 10,

$$4 + 6 + 3 = 13$$

$$4 + 9 = 13$$

So that 13 - 4 = 9.

Rewriting a subtraction equation in a different form can also be helpful for the problem 8 - ☐ = 2 in task 1:11. Equations of this form can arise when analyzing word problems with situation type "Take From with Change Unknown" (see Teacher Notes for task **1:13 Falling Icicles**). Thinking of 8 as decomposed into two parts, one part equal to 2, the equation 8 - ☐ = 2 asks for the other part—a problem that can be restated as 2 + ☐ = 8. Now the unknown addend could be found by a Level 2 method, or simply thought of as 8 - 2, that is, 6. Similarly, the problem ☐ - 5 = 4 in task 1:11 could be rewritten as 4 + 5 = ☐, which leads to the answer 9.

Finally, another way to find a difference by using the relationship between addition and subtraction is simply to know the relevant decomposition. For example, in the problem 6 + ☐ = 12 from task 1:11, some or many students by end of grade 1 might simply remember the "doubles fact" 6 + 6 = 12 and thus quickly know that the unknown addend is 6. Remembering all the single-digit sums and fluently finding the related differences isn't a grade 1 expectation (CCSS 2.OA.B.2), but some or many grade 1 students will gain enough experiences during the year to know a few, or more than a few decompositions from memory by the end of the year. If students do find a difference $C - A$ by remembering the related sum $A + B = C$, they don't need to explain or justify the answer beyond simply "I knew that $A + B = C$." But they should be able to discuss and understand another

Additional notes on the design of the task

- In the left-hand column, the last problem features an addition sign on both sides of the equal sign. This emphasizes number sense and the properties of addition as well as the meaning of the equal sign (CCSS 1.OA.D.7).
- The task includes the direction, "Tell how you got the answers" so that students can produce mathematical language, and so as to reveal the mix of Level 1, Level 2, and Level 3 methods at work in students' thinking.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:11? In what specific ways do they differ from 1:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

student's Level 2 or Level 3 method as the occasion to do so arises. No student needs to use all the methods described in the *Progression* document, but the methods all have mathematical value, and students should be able to discuss and understand each other's methods.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting on; finding partners of 10 and partners of smaller numbers; using the structure of teen numbers; and using the relationship between addition and subtraction.

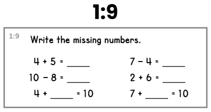
Extending the task

How might students drive the conversation further?

- Students could choose one of the equations in task 1:11 and create a word problem that corresponds to it.
- If a student and a partner used different methods for one of the problems in task 1:11, the student and partner could switch and try each other's method.

Related Math Milestones tasks

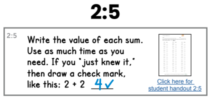
1:9


1:9 Write the missing numbers.

4 + 5 = ____ 7 − 4 = ____
10 − 8 = ____ 2 + 6 = ____
4 + ____ = 10 7 + ____ = 10

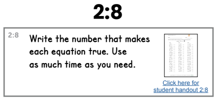
Task **1:9 Fluency within 10** concentrates on problems within the grade 1 fluency goal (CCSS 1.OA.C.6). The word problems in grade 1 involve addition and subtraction within 20; see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

2:5


2:5 Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2 4✓

Click here for student handout 2:5

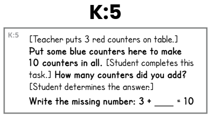
2:8


2:8 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 2:8

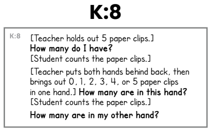
In later grades, tasks **2:5 Sums of Single-Digit Numbers** and **2:8 Fluency within the Addition Table** represent the culminating fluency and remembering goals for addition and subtraction within 20 (CCSS 2.OA.B.2).

K:5


K:5 [Teacher puts 3 red counters on table.] **Put some blue counters here to make 10 counters in all.** [Student completes this task.] **How many counters did you add?** [Student determines the answer.]

Write the missing number: 3 + ____ = 10

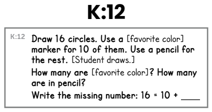
K:8


K:8 [Teacher holds out 5 paper clips.] **How many do I have?** [Student counts the paper clips.]

[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?** [Student counts the paper clips.]

How many are in my other hand?

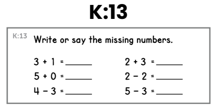
K:12


K:12 **Draw 16 circles. Use a** [favorite color] **marker for 10 of them. Use a pencil for the rest.** [Student draws.]

How many are [favorite color]**? How many are in pencil?**

Write the missing number: 16 = 10 + ____

K:13


K:13 Write or say the missing numbers.

3 + 1 = ____ 2 + 3 = ____
5 + 0 = ____ 2 − 2 = ____
4 − 3 = ____ 5 − 3 = ____

In earlier grades, task **K:5 Adding to Make a Group of Ten** involves finding a partner of 10. Task **K:8 Five Behind the Back** involves decompositions of 5, and task **K:12 Make Ten and Some More** involves the structure of teen numbers. Task **K:13 Fluency within Five** concentrates on problems within the kindergarten fluency goal (CCSS K.OA.A.5).

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in this Teaching Note refer to this *Progression* document.

‡ See also "How I see addition facts," (Zimba, 2016).

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:12 Blowing Out Candles

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: ______________

Answer: _____ candles are still lit.

Answer

15 - 9 = ?, 15 = 9 + ?, or another equivalent equation. 6 candles are still lit.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

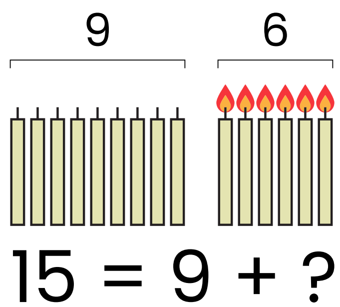
Concepts, Application

Additional notes on the design of the task

If students are unfamiliar with the situation of blowing out candles on a cake, the situation could be explained, for example, by showing a video clip.

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

The situation in task 1:12 can be thought of as "Put Together/Take Apart with One Addend Unknown."‡ That is, the 15 candles can be viewed as composed of a group of unlit candles and a group of lit candles. There are 9 unlit candles, and the number of lit candles is initially unknown.



During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

Task 1:12 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("6 candles"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between addition and subtraction.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document (p. 13) stresses that "[i]f textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:12? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:12? In what specific ways do they differ from 1:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Some equation models describe a situation in an algebraic way, such as 15 = 9 + ? (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as 15 - 9 = ? (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Word problems vary considerably in the uses to which they put addition and subtraction, and they also vary in the complexity of the calculation required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. The calculation in task 1:12 involves calculating the difference 15 - 9. Calculations of this kind are new to students in grade 1.

The difference 15 - 9 could be calculated in many ways; see the section on "Using Level 2 and Level 3 strategies to extend addition and subtraction problem solving beyond 10, to problems within 20," pp. 14–17 in the *Progression* document. An example of a Level 2 method would be counting on; paraphrasing the *Progression* document, this approach involves seeing the 9 as part of 15, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9: thus the student may say,

"*Niiiiine, ten, eleven, twelve, thirteen, fourteen, fifteen*."

1 2 3 4 5 6

Students keep track of how many they counted on (here, 6) with fingers, mental images, or physical actions such as head bobs. Elongating the first counting word ("Niiiiine…") is natural and indicates that the student differentiates between the first addend and the counts for the second addend.

Meanwhile, an example of a Level 3 method would be to think that

$$\begin{aligned} 15 &= 10 + 5 \\ &= 9 + 1 + 5 \\ \text{so } 15 &= 9 + 6, \end{aligned}$$

that is, 15 - 9 = 6. Alternatively,

$$\begin{aligned} 15 - 9 &= 15 - 5 - 4 \\ &= 10 - 4 \\ &= 6 \end{aligned}$$

a method that depends on knowing decompositions of 9 and of 10. Students for whom the calculation 15 - 9 is time-consuming and/or effortful may need to be redirected to the context after obtaining the result 15 - 9 = 6, so as to relate the numbers in this equation to the context and answer the question in task 1:12.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two one-digit numbers with sum greater than 10, and performing a related subtraction; writing equations to describe quantitative relationships; and fundamental concepts of addition and subtraction.

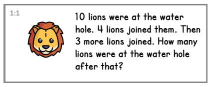
Extending the task

How might students drive the conversation further?

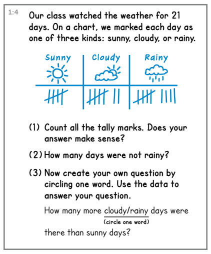
- For each number in an equation model for the situation, students could identify the quantity in the situation to which the number refers. For example, 9 refers to "the number of candles that are still lit." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could ask or answer additional questions about the situation, such as "If Grace tries a second time and blows out every candle but one, how many candles did Grade blow out the second time?"

Related Math Milestones tasks

1:1


1:1
10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

1:4


1:4
Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

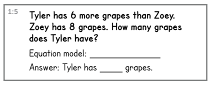
(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy (circle one word) days were there than sunny days?

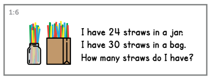
1:5


1:5
Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

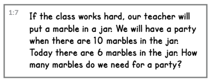
Equation model: ______________

Answer: Tyler has _____ grapes.

1:6


1:6
I have 24 straws in a jar.
I have 30 straws in a bag.
How many straws do I have?

1:7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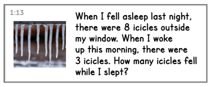
If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

1:13


1:13
When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Besides task 1:12, other word problems and their situation types in grade 1 are as follows: tasks **1:1 Lions at the Watering Hole**, *Add To with Result Unknown* (two-step); **1:4 Analyzing Weather Data**, *Put Together/Take Apart with Total Unknown* (part (2)) *and Compare with Difference Unknown ('how many more' language)* (part (3)); **1:5 Tyler's Grapes**, *Compare with Bigger Quantity Unknown ('more' language)*; **1:6 Two Groups of Straws**, *Put Together/Take Apart with Total Unknown*; **1:7 Class Marble Jar**, *Add To with Change Unknown*; and **1:13 Falling Icicles**, *Take From with Change Unknown.*

In earlier and later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

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1:13 Falling Icicles

Teacher Notes

Math Milestones™

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Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

1:13

When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Answer

5 icicles fell while I slept.

Click here for a student-facing version of the task.

Refer to the Standards

1.OA.A.1, 1.OA; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts, Application

Additional notes on the design of the task

The photo in the task shows 8 icicles, which allows students to discuss the situation in relation to the photo. The photo also clarifies the context for students who may be unfamiliar with cold weather.

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation in task 1:13 can be thought of as "Take From with Change Unknown."[‡] Some icicles were 'taken away,' but initially it is unknown how many icicles were taken away. Thus, an equation model for the situation could be written as 8 - ☐ = 5. This situation equation (p. 13) could then be rewritten as a solution equation, 8 - 5 = ☐.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for calculating with fractions are different from the algorithms for calculating with base-ten numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. The mathematical relationship between addition and subtraction also remains the same regardless of what kinds of numbers (or variables) are involved: specifically, $C - A$ is the unknown addend in A + ☐ = C. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding. In terms of the situation in task 1:13, the number of icicles that fell can be found by subtraction, 8 - 3 = 5, and 5 can also be seen as an addend: the number of icicles that would have to 'put back up' in order to restore the original number: 3 + 5 = 8.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: subtracting two one-digit numbers; adding two one-digit numbers; and fundamental concepts of addition and subtraction.

Extending the task

How might students drive the conversation further?

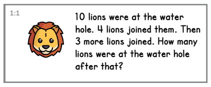
- Students could reinterpret the situation along the lines of tasks **K:2 Two Groups of Books** or **1:6 Two Groups of Straws** (*Put Together/ Take Apart with Total Unknown*) by drawing two groups of icicles: one group of 3 icicles hanging by the window, and one group of 5 icicles lying on the ground. This view of the situation may help to relate the equations 8 - 3 = ? and 3 + 5 = 8.
- Students could create their own word problem to represent 8 - 3 = ?.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:13? In what specific ways do they differ from 1:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

1:1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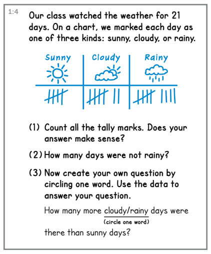
1:4


1:4
Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

(1) Count all the tally marks. Does your answer make sense?

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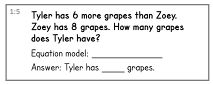
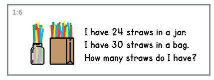
1:5


1:5
Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: ________________

Answer: Tyler has _____ grapes.

1:6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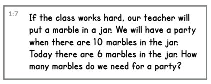
1:7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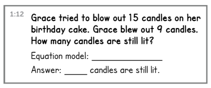
1:12


1:12
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- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

1:14 Shape True/False

Teacher Notes

Central math concepts

As explained in the *Progression* document (p. 2),[†] the three themes of elementary-grades geometry are:

- Composing and decomposing shapes;
- Reasoning with shape components, shape properties, and shape categories; and
- Spatial relations and spatial structuring.

These three themes are involved in the three statements that students evaluate in task 1:14.

To verify the first statement, two of the triangles can be composed to make a rectangle, and then the rectangles can be used as a new unit for composing a square, as shown in the figure that appears in the Answer section. Thus, "With experience, students are able to use a composed shape as a new unit in making other shapes" (p. 4). One reason this is important is that "there is suggestive evidence that this type of composition corresponds with, and may support, children's ability to compose and decompose numbers" (p. 3).

The second statement involves shape attributes and categories. The G Progression document (p. 3) describes three *levels of geometric thinking* that describe increasing sophistication with this learning progression:

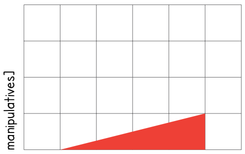
- **Visual/Syncretic level.** Students recognize shapes, for example, a rectangle "looks like a door."
- **Descriptive level.** Students perceive properties of shapes, for example, a rectangle has four sides, all its sides are straight, opposite sides have equal length.
- **Analytic level.** Students characterize shapes by their properties, for example, a rectangle has opposite sides of equal length and four right angles.
- **Abstract level.** Students understand, for example, that a rectangle is a parallelogram because it has all the properties of parallelograms.

Identifying the second statement as false can involve a mix of Syncretic reasoning (the pentagon "doesn't look square") and Descriptive reasoning (even though the square is oriented diagonally, it still has the properties of a square) (p. 8).

A recurrent question in mathematics concerns what it means for two objects of a certain kind to count as "the same."[‡] In later-grades geometry for example, congruence defines what it means for two distinct figures to count as "the same" despite differences of position or orientation. To a young student, a square oriented diagonally looks different from a square with its sides oriented with the edges of

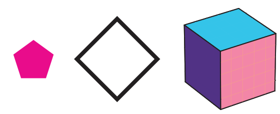
1:14 One statement below is false. Find the false statement. How did you decide?

[Student uses manipulatives]

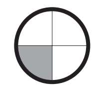


A square can be created using triangles like this one.

None of these are squares.

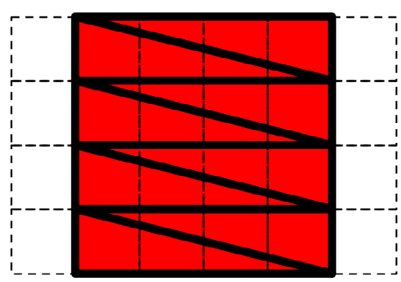


The shaded part of the circle is one fourth of the whole circle.



Answer

The false statement is "None of these are squares." Explanations may vary. Identifying the first statement as true involves composing the triangles to make a square, as shown in the figure. Identifying the second statement as false relies on recognizing that the second shape in the corresponding group of three shapes is, indeed, a square. Identifying the third statement as true relies on recognizing that four of the shaded parts make up 1 whole circle.



Click here for a student-facing version of the task.

the paper—reasonably enough, since with respect to orientation at least, the diagonal square *is* different. Likewise, there is a literal sense in which a red square and a blue square *are* different, at least with respect to color. Students aren't wrong to perceive these differences. But the progression from Visual/Syncretic toward Descriptive involves coming to use the category "square" in mathematical discourse in such a way that orientation, color, and overall size "no longer count."

Levels of thinking in spatial structuring

Levels of thinking portrayed by different students as they attempted to complete a drawing of an array of squares, given one column and row. This was an assessment, not an instructional task.

In the third statement, a circle is spatially structured into congruent parts. Spatial structuring is "the mental operation of constructing an organization or form for an object or set of objects in space," and it builds on students' experiences with shape composition (p. 4). Such spatial structuring will be used in grade 3 to understand area measurement for rectangles and in grade 5 to understand volume measurement for right rectangular prisms. Spatial structuring is also involved in partitioning of wholes during division and fraction reasoning in grades 3–6. Thus, "spatial structuring precedes meaningful mathematical use of the structures" (p. 4). The figure shows levels of spatial structuring portrayed by different students in response to an assessment task (p. 11).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: recognizing and naming shapes; composing triangles and rectangles; and identifying and naming shape attributes.

Refer to the Standards

1.G.A; MP.1, MP.3, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor

Concepts

Additional notes on the design of the task

The first statement is intended to be explored using manipulatives. The second statement could also be posed using manipulatives (flat pentagon, flat square, and solid cube).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 1:14? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 1:14? In what specific ways do they differ from 1:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 1:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

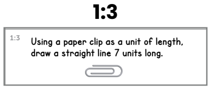
Extending the task

How might students drive the conversation further?

- With reference to the first statement, if students compose a square, they could state the properties that convince them they have in fact made a square instead of a (non-square) rectangle.
- With reference to the second statement, if students correctly explain that the statement is false "because the second shape is a square," they could continue the discussion by explaining why the first and third shapes (the pentagon and the cube) *aren't* squares. Press for precision using attribute language: for example, a sufficient reason why the pentagon isn't a square is that it has five sides, not four. (This is a more robust explanation than simply saying that the pentagon isn't a square "because it's a pentagon.")
- With reference to the third statement, students could compose two of the fourths and describe the resulting shape as half of the circle.

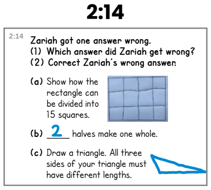
Related Math Milestones tasks

1:3

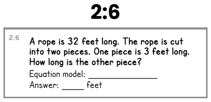


Spatial structuring and composing/decomposing are involved in length measurement (iterating length units), as in task **1:3 Paper Clip Length Units**.

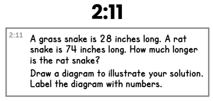
2:14



2:6

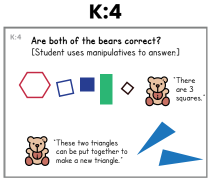


2:11



In later grades, task **2:14 Correcting a Shape Answer** involves the same three themes of elementary grades geometry as task 1:14. Composing and decomposing is involved in adding and subtracting lengths, as in tasks **2:6 Cutting a Rope** and **2:11 Grass Snake vs. Rat Snake**.

K:4



In earlier grades, task **K:4 Bears Talk About Shapes** involves shape attributes and composing shapes.

† Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression*.

‡ In algebra, there are two important questions along these lines: (1) What does it mean for two different-looking expressions to be equivalent? (2) What does it mean for two different-looking functions to be the same function?

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

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