2:11 Grass Snake vs. Rat Snake

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

For addition and subtraction specifically, the mathematical relationship between addition and subtraction is that $C - A$ is the unknown addend in $A + ? = C$. (One might paraphrase this statement by saying that, "Given a total and one part, subtraction finds the other part.") Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the

2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?

Draw a diagram to illustrate your solution. Label the diagram with numbers.

Answer

The rat snake is 46 inches longer than the grass snake. Diagrams may vary but might take the form of a tape diagram (appropriately labeled with the values 28 and 74 and the length difference value 46); a number line diagram (with locations indicated for the numbers 0, 28, and 74 and with the interval between 28 and 74 labeled with 46); or a more or less abstract rendering of the snakes themselves with appropriate labeling of lengths and the length difference.

Click here for a student-facing version of the task.

Refer to the Standards

2.MD.B.5, 2.MD.B; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application, Procedural skill and fluency

Additional notes on the design of the task

- The task does not include an illustration, because creating a diagram is part of the task.

mathematical relationship between addition and subtraction can play out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation type in task 2:11 is called "Compare with Difference Unknown."‡

As noted in the *Progression* document, "One reason Compare problems are more advanced than the other two major types is that in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the 'extra' that when added to the smaller unknown makes the total equal to the bigger unknown or by finding this quantity embedded within the bigger unknown" (p. 12).

The context in task 2:11 sets up a comparison of two length measurements. Thus, the "objects" being counted in this problem are length units. This situation contrasts with Compare problems involving more tangible objects such as apples or grapes. It is important to include length contexts in grade 2 word problems, because relating addition and subtraction to length, including by working with number lines, is part of early preparation for working with fractional quantities in grades 3 and later.

Task 2:11 asks not only for the final answer but also for a diagram. A diagram is requested because compared to the answer alone ("46 inches"), a diagram is better evidence that students have comprehended the situation and its quantitative relationships. The diagram records the situation's mathematical structure so that students can discuss and reflect on it. The diagram can also illustrate the relationship between addition and subtraction.

Not all students will create the same diagram to illustrate their solution. The *Progression* document (p. 13) stresses that "If textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Word problems vary considerably in the uses to which they put addition and subtraction, and they also vary in the complexity of the calculation required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. The calculation in task 2:11 involves calculating a difference of two two-digit numbers in a case that also involves regrouping. Calculations of this kind are among the grade 2 fluency targets, but they are also new to students in this grade. Students for whom the calculation 74 - 28 is time-consuming and/or effortful may need to be redirected to the context after obtaining the result 74 - 28 = 46, so

Additional notes on the design of the task (continued)

- Grass snake and rat snake are real snake species, and the two individuals described in the task have reasonable lengths for their species. Students may not have heard of either species, however. After working on the task, some students might be interested to discuss the two kinds of snake.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 2:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 2:11? In what specific ways do they differ from 2:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 2:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ For the other situation types, see Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona). The *Progression* document has extensive discussion of aspects of teaching and learning about Compare problems. Note that Compare problems are studied in Grades 1 and 2; quotations from the *Progression* document are taken from the section for Grade 1, because Compare problems are discussed there first.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

as to relate the numbers in this equation to the context and answer the question in the task.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: calculating sums of two two-digit numbers; writing equations to describe quantitative relationships; and fundamental concepts of multiplication with whole numbers.

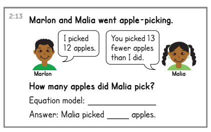
Extending the task

How might students drive the conversation further?

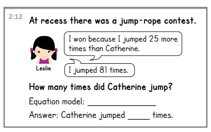
- Students with an interest in the topic could compare the lengths of the snakes in the task to typical lengths of some other snake species they know about, such as a cobra, anaconda, or even a prehistoric snake such as Titanoboa or Sanajeh. (The lengths of the snakes could be provided in inches.)

Related Math Milestones tasks

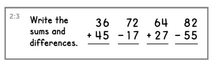
2:13



2:12

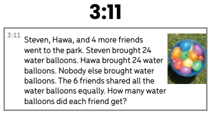


2:3

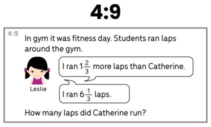


Task **2:13 Apple-Picking** is a word problem of situation type *Compare with Bigger Unknown*. Task **2:12 Jump-Rope Contest** is a word problem of situation type *Compare with Smaller Unknown*. Task **2:3 Fluency within 100 (Add/Subtract)** is a fluency task involving two-digit sums and differences.

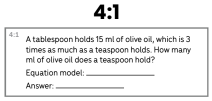
3:11



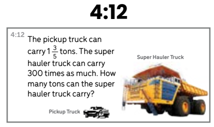
4:9



4:1

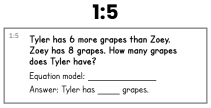


4:12

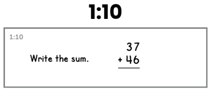


In later grades, task **3:11 Water Balloons** is a multi-step word problem involving more than one operation. Task **4:9 Fitness Day** is a word problem of situation type *Compare with Smaller Unknown* that involves fraction quantities. Tasks **4:1 A Tablespoon of Oil** and **4:12 Super Hauler Truck** are also comparison problems, but the comparisons in these tasks are multiplicative, not additive.

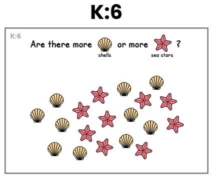
1:5



1:10



K:6



In earlier grades, task **1:5 Tyler's Grapes** is a word problem of situation type *Compare with Smaller Unknown*. Task **1:10 Two-Digit Addition** involves finding the sum of two two-digit numbers. Task **K:6 More Shells or More Stars?** involves comparison but without finding how many more.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?