2:13 Apple-Picking

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

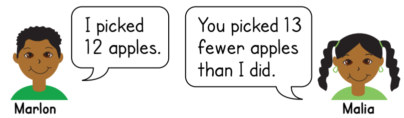
1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

For addition and subtraction specifically, the mathematical relationship between addition and subtraction is that $C - A$ is the unknown addend in $A + ? = C$. (One might paraphrase this statement by saying that, "Given a total and one part, subtraction finds the other part.") Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the

2:13 Marlon and Malia went apple-picking.



How many apples did Malia pick?

Equation model: ______________

Answer: Malia picked _____ apples.

Answer

? - 13 = 12, 12 + 13 = ?, or another equivalent equation. Malia picked 25 apples.

Click here for a student-facing version of the task.

Refer to the Standards

2.OA.A.1; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

Word problems involving Compare situations can sometimes consist of complex text. Therefore, task 2:13 presents the given information in the form of a dialogue between Marlon and Malia. This could also invite an approach of having students convey the task to each other by acting out the dialogue.

mathematical relationship between addition and subtraction can play out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation type in task 2:13 is called "Compare with Bigger Unknown." It is a Compare situation because Malia is using subtraction to compare her 25 apples with Marlon's 12 apples; and more specifically, the situation is "Compare with Bigger Unknown" because the initially unknown quantity is how many apples Malia has (and Malia has the bigger apple haul).

For the other situation types, see [Table 2, p. 9](#) of the *Progression* document.[‡] It has extensive discussion of aspects of teaching and learning about Compare problems,[§] including the following points ([pp. 12, 13](#)).

- "One reason Compare problems are more advanced than the other two major types is that in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the "extra" that when added to the smaller unknown makes the total equal to the bigger unknown or by finding this quantity embedded within the bigger unknown."
- "The language of comparisons is also difficult. For example, 'Julie has three more apples than Lucy' tells both that Julie has more apples and that the difference is three. Many students 'hear' the part of the sentence about who has more, but do not initially hear the part about how many more; they need experience hearing and saying a separate sentence for each of the two parts in order to comprehend and say the one-sentence form."
- "Another language issue is that the comparing sentence might be stated in either of two related ways, using more or less. Students need considerable experience with less to differentiate it from more; some children think that less means more."

Task 2:13 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("25 apples"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between addition and subtraction.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document ([p. 13](#)) stresses that "[i]f textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 2:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 2:13? In what specific ways do they differ from 2:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 2:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report.* San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

§ Compare problems are studied in grades 1 and 2. Quotations from the *Progression* document are taken from the section for grade 1 because Compare problems are first discussed there.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Some equation models describe a situation in an algebraic way, such as ? - 12 = 13 (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as ? = 12 + 13 (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: calculating sums of two two-digit numbers; writing equations to describe quantitative relationships; and fundamental concepts of addition and subtraction.

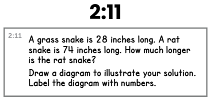
Extending the task

How might students drive the conversation further?

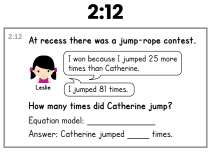
- For each number in an equation model for the situation, students could identify the quantity in the situation to which the number refers. For example, 25 refers to "the number of apples Malia picked." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could add a third person to the story who outdoes both Marlon and Malia. Students could create new dialogue and ask a new question. For example, new dialogue could say, "Oh yeah? I picked 4 more apples than Malia," and a new question could be, "How many more apples than Marlon did the new person pick?"

Related Math Milestones tasks

2:11

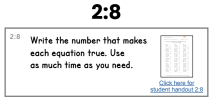

2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?
Draw a diagram to illustrate your solution. Label the diagram with numbers.

2:12


2:12 At recess there was a jump-rope contest.

How many times did Catherine jump?
Equation model: ______________
Answer: Catherine jumped _____ times.

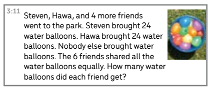
2:8


2:8 Write the number that makes each equation true. Use as much time as you need.
Click here for student handout 2:8

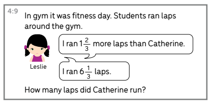
Task **2:11 Grass Snake vs. Rat Snake** is a word problem of situation type *Compare with Difference Unknown* that relates addition and subtraction to length. Task **2:12 Jump-Rope Contest** is a word problem of situation type *Compare with Smaller Unknown*. Task **2:8 Fluency within the**

Addition Table is a fluency task involving the relationship between addition and subtraction. See also the [Map of Addition and Subtraction Situations in K–2 Math Milestones](#).

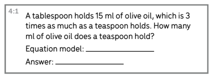
3:11



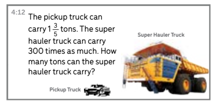
4:9



4:1

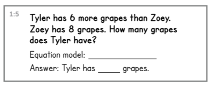


4:12

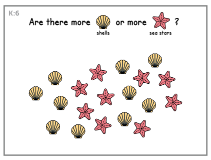


In later grades, task **3:11 Water Balloons** is a multi-step word problem involving more than one operation. Task **4:9 Fitness Day** is a word problem of situation type *Compare with Smaller Unknown* that involves fraction quantities. Tasks **4:1 A Tablespoon of Oil** and **4:12 Super Hauler Truck** are also comparison problems, but the comparisons in these tasks are multiplicative, not additive.

1:5



K:6



In earlier grades, task **1:5 Tyler's Grapes** is a word problem of situation type *Compare with Smaller Unknown*. Task **K:6 More Shells or More Stars?** involves comparison but without finding how many more.

2:13 Apple-Picking

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?