2:14 Correcting a Shape Answer

Teacher Notes

Math Milestones™

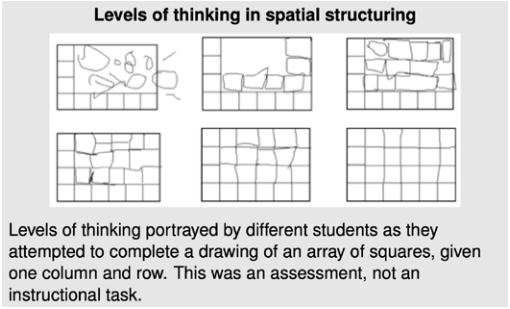
Central math concepts

As explained in the *Progression* document (p. 2),† the three themes of elementary-grades geometry are:

- Spatial relations and spatial structuring;
- Composing and decomposing shapes; and
- Reasoning with shape components, shape properties, and shape categories.

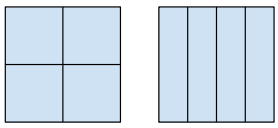
These three themes are involved in the three parts (a), (b), and (c) of task 2:14.

In part (a), a rectangle is spatially structured into an array of squares. Spatial structuring is "the mental operation of constructing an organization or form for an object or set of objects in space," and it builds on students' experiences with shape composition (p. 4). Such spatial structuring will be used in grade 3 to understand area measurement for rectangles, and in grade 5 to understand volume measurement for right rectangular prisms. Spatial structuring is also involved in partitioning of wholes during division and fraction reasoning in grades 3–6. Thus, "spatial structuring precedes meaningful mathematical use of the structures" (p. 4). The figure shows levels of spatial structuring portrayed by different students in response to an assessment task (p. 11).



Levels of thinking portrayed by different students as they attempted to complete a drawing of an array of squares, given one column and row. This was an assessment, not an instructional task.

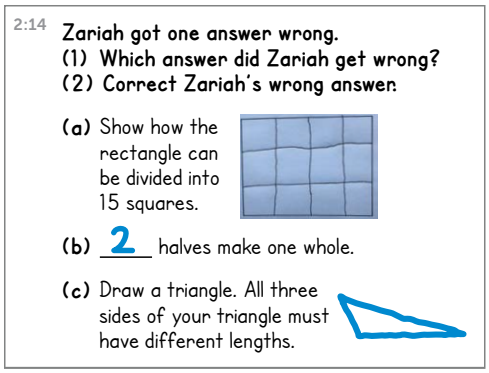
In part (b), two halves are composed to make one whole. This part of task 2:14 is linguistic in nature, but it draws on students' experiences with shape composition, as in the figure for example. Composing into a whole, and decomposing into halves and fourths, also prepares for fraction work in grade 3.



Decomposing a square into fourths in two different ways. A fourth on the left can be shown to be equal in size (equal in area) to a fourth on the right by decomposing it into parts and composing the parts into a different shape.

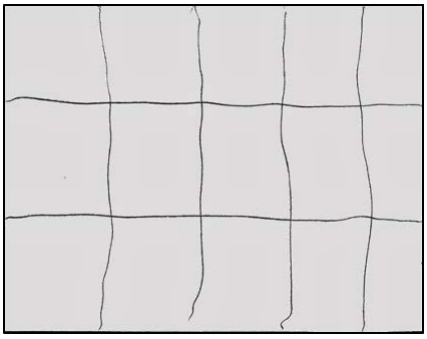
In part (c), a triangle is discussed in terms of its attributes. The *Progression* document (p. 3) describes three *levels of geometric thinking* that describe increasing sophistication with this learning progression:

- **Visual/Syncretic level**. Students recognize shapes, for example, a rectangle "looks like a door."
- **Descriptive level**. Students perceive properties of shapes, for example, a rectangle has four sides, all its sides are straight, opposite sides have equal length.



Answer

(1) Answer (a) is wrong. **(2)** See the example of a corrected answer for (a).



Click here for a student-facing version of the task.

Refer to the Standards

2.G.A; MP.1, MP.3, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

- **Analytic level**. Students characterize shapes by their properties, for example, a rectangle has opposite sides of equal length and four right angles.
- **Abstract level**. Students understand, for example, that a rectangle is a parallelogram because it has all the properties of parallelograms.

Task 2:14 connects properties of shapes at the Analytic level with students' focus on length measurement in this grade.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: composing and decomposing squares and rectangles; identifying and naming shape attributes; and reasoning about length and/or measuring length.

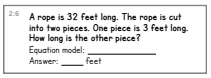
Extending the task

How might students drive the conversation further?

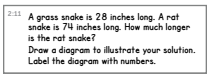
- Students could extend part (b) by drawing shapes that illustrate 2 halves making 1 whole, aiming for variety in the resulting drawings.
- Students could extend further by drawing shapes that illustrate 4 fourths making 1 whole, aiming for variety in the resulting drawings.

Related Math Milestones tasks

2:6

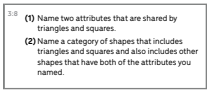

2:6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?
Equation model: _____
Answer: ____ feet

2:11

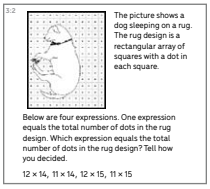

2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?
Draw a diagram to illustrate your solution. Label the diagram with numbers.

Spatial structuring and composing/decomposing are involved in length measurement (iterating length units), and composing and decomposing is involved in adding and subtracting lengths, as in tasks **2:6 Cutting a Rope** and **2:11 Grass Snake vs. Rat Snake**.

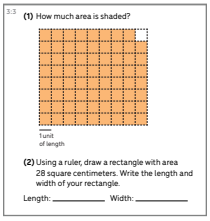
3:8


3:8 (1) Name two attributes that are shared by triangles and squares.
(2) Name a category of shapes that includes triangles and squares and also includes other shapes that have both of the attributes you named.

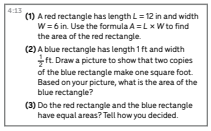
3:2


3:2 The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.
Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.
12 × 14, 11 × 14, 12 × 15, 11 × 15

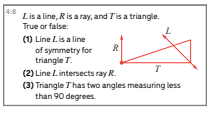
3:3


3:3 (1) How much area is shaded?
Unit of length
(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.
Length: _____ Width: _____

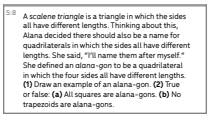
4:13


4:13 (1) A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.
(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?
(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

4:8


4:8 L is a line, R is a ray, and T is a triangle.
True or false:
(1) Line L is a line of symmetry for triangle T.
(2) Line L intersects ray R.
(3) Triangle T has two angles measuring less than 90 degrees.

5:8


5:8 A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths.
(1) Draw an example of an alana-gon. (2) True or false: (a) All squares are alana-gons. (b) No trapezoids are alana-gons.

In later grades, task **3:8 Shape Attributes and Categories** involves reasoning with shape components, shape properties, and shape categories. Spatial structuring is involved in tasks **3:2 Hidden Rug Design**, **3:3 Length and Area Quantities**, and **4:13 Area Units**. Task **4:8 Shapes with Given Positions** involves definitions and properties of geometric figures, and task **5:8 Alana's New Shape Category** involves an invented category defined by attributes.

Additional notes on the design of the task

Correcting an error is one way for primary-grades students to engage in Standard for Mathematical Practice MP.3, "Construct viable arguments and critique the reasoning of others." (CCSS MP.3)

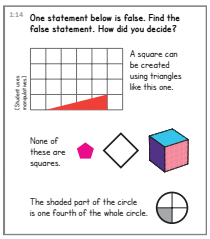
Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 2:14? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 2:14? In what specific ways do they differ from 2:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 2:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

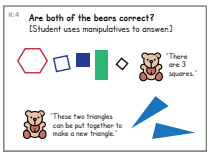
† Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression*.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

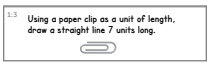
1:14



K:4



1:3



In earlier grades, task **1:14 Shape True/False** involves the same three themes of elementary-grades geometry as task 2:14. In earlier grades, task **K:4 Bears Talk About Shapes** involves shape attributes and composing shapes. Spatial structuring and composing/decomposing are involved in length measurement (iterating length units), as in task **1:3 Paper Clip Length Units**.

2:14 Correcting a Shape Answer

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?