

# 2:1 Paper Chain

## Teacher Notes



### Central math concepts

Based on interviews with middle-grades students,<sup>†</sup> education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will “tell” you which operation to use (e.g., “...if it’s like, 78 and maybe 54, then I’d probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers”).
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated “key” words or phrases to tell which operations to use (e.g., “all together” means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder’s list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to “tell” them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

In particular, task 2:1 involves addition and subtraction and the relationship between them. The mathematical relationship between addition and subtraction is that  $C - A$  is the unknown addend in  $A + ? = C$ . Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

From an abstract point of view, there’s not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between addition and subtraction can play

2:1 Avi made a paper chain. Then Avi added 29 more links to the paper chain. Now there are 52 links in the paper chain. How many links were in the paper chain before?



### Answer

There were 23 links in the paper chain before.

[Click here](#) for a student-facing version of the task.

### Refer to the Standards

2.OA.A.1, 2.NBT.B.5; MP.1, MP.2, MP.4.

Standards codes refer to [www.corestandards.org](http://www.corestandards.org).

One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

### Aspect(s) of rigor:

Concepts, Application, Procedural skill and fluency

### Additional notes on the design of the task

- The calculation involved in the task,  $52 - 29$ , is a two-digit addition problem which belongs to the fluency target in grade 2; see [CCSS 2.NBT.B.5](#) and the [Teacher Notes](#) for task **2:3 Fluency within 100 (Add/Subtract)**.
- The first sentence, “Avi made a paper chain,” does not refer explicitly to one of the quantities in the situation, which is the number of links that were in the paper chain to begin with. Nevertheless, as soon as the chain was made, that quantity was present in the situation. The quantity is eventually named explicitly by the last sentence in the task (the sentence ending with a question mark).

out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:<sup>†</sup>

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 2:1 is called “Add To with Start Unknown.” It is an Add To situation because Avi adds links to the chain; and more specifically, the situation is “Add To with Start Unknown” because the initially unknown quantity is how many links were in the chain to begin with.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are extended into working with fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

Word problems vary considerably in the uses to which they put addition and subtraction, and they also vary in the complexity of the calculation required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. The calculation in task 2:1 involves calculating a difference of two two-digit numbers in a case that also involves regrouping. Calculations of this kind are new to students in grade 2; students work toward fluency with these calculations throughout the year. Students for whom the calculation  $52 - 29$  is time-consuming and/or effortful may need to be redirected to the context after obtaining the result  $52 - 29 = 23$ , so as to relate the numbers in this equation to the context and answer the question in the task.

The difference  $52 - 29$  could be calculated in many ways using place value, properties of operations, and/or the relationship between addition and subtraction. A student could use a pencil and paper method based on tens and ones (see [Teacher Notes](#) for task **2:3 Fluency within 100 (Add/Subtract)**). A student could calculate mentally in ways such as in the following examples:

- $52 - 29 = 52 - 30 + 1 = 22 + 1 = 23$ .
- $29 + 1 = 30$ ,  $30 + 20 = 50$ ,  $50 + 2 = 52$ , so  $29 + 23 = 52$ .

Such mental calculation methods should be discussed and compared in connection with corresponding written equations.

## Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 2:1? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 2:1? In what specific ways do they differ from 2:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 2:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?\*

<sup>†</sup> Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. <https://files.eric.ed.gov/fulltext/ED290629.pdf>

<sup>‡</sup> See [Table 2, p. 9](#) of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

\* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.



## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding and/or subtracting with two-digit numbers; and working with the count sequence within 100.

## Extending the task

How might students drive the conversation further?

- Students could represent the quantities in the situation in two ways, by creating a diagram (such as a tape diagram, for example) and by creating a situation equation (such as  $? + 29 = 52$ ), then identify correspondences between the equation and the diagram.
- Students could check their difference by adding.
- Students could imagine that the chain got torn in two. Assuming one link was ruined and thrown away, then what are two possible numbers for the number of links in the two smaller chains?



## Related Math Milestones tasks

<p><b>2:6</b></p> <p>2.6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece? Equation model: _____ Answer: _____ feet</p>	<p><b>2:9</b></p> <p>2.9 A farmer said, "Last night some deer came and ate 16 of my cabbages. Now I only have 38 cabbages." How many cabbages were there before the deer came? Equation model: _____ Answer: There were _____ cabbages.</p>	<p><b>2:11</b></p> <p>2.11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake? Draw a diagram to illustrate your solution. Label the diagram with numbers.</p>	<p><b>2:12</b></p> <p>2.12 At recess there was a jump-rope contest. I won because I jumped 25 more times than Catherine. Laska jumped 81 times. How many times did Catherine jump? Equation model: _____ Answer: Catherine jumped _____ times.</p>
<p><b>2:13</b></p> <p>2.13 Marlon and Malia went apple-picking. I picked 12 apples. You picked 13 fewer apples than I did. How many apples did Malia pick? Equation model: _____ Answer: Malia picked _____ apples.</p>	<p><b>2:4</b></p> <p>2.4 Faith went to the park. The picture graph shows all of the animals Faith saw. 1 cow, 3 horses, 1 butterfly, 1 squirrel. Faith said, "I saw fewer butterflies than birds." How many fewer butterflies did Faith see?</p>		

Other word problems and their situation types in grade 2 are as follows: tasks **2:6 Cutting a Rope**, *Put Together with One Addend Unknown*, in a context involving length; **2:9 Disappearing Cabbages**, *Take From with Start Unknown*; **2:11 Grass Snake vs. Rat Snake**, *Compare with Difference Unknown* ('how many more'/'how much longer' language), in a context involving length; **2:12 Jump-Rope Contest**, *Compare with Smaller Quantity Unknown* ('more' language); **2:13 Apple-Picking**, *Compare with Bigger Quantity Unknown* ('fewer' language); and **2:4 Animals in the Park**, combination of *Put Together/Take Apart with Total Unknown* and *Compare with Difference Unknown* ('how many fewer' language).

<p><b>4:9</b></p> <p>4.9 In gym it was fitness day. Students ran laps around the gym. I ran 12 more laps than Catherine. I ran <math>6\frac{1}{2}</math> laps. How many laps did Catherine run?</p>	<p><b>5:9</b></p> <p>5.9 On Saturday there was a walkathon. I walked <math>\frac{1}{2}</math> mile farther than Leslie. Catherine walked <math>\frac{1}{4}</math> mile. How many miles did Leslie walk?</p>
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In later grades, tasks **4:9 Fitness Day** and **5:9 Walkathon** are addition/subtraction word problems of situation type *Compare with Smaller Quantity Unknown*.

In earlier grades, see the [Map of Addition and Subtraction Situations in K–2 Math Milestones](#).



### Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions. 

#### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

#### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

#### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?