2:6 Cutting a Rope

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

In particular, task 2:6 involves addition and subtraction and the relationship between them. The mathematical relationship between addition and subtraction is that $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between addition and subtraction can play

2:6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?

Equation model: ______________

Answer: _____ feet

Answer

3 + ? = 32, 32 - 3 = ?, or another equivalent equation, or an equation showing the value of the difference or unknown addend, such as 3 + 29 = 32, 32 - 3 = 29, or another equivalent equation. The other piece is 29 feet long.

Click here for a student-facing version of the task.

Refer to the Standards

2.MD.B.5, 2.MD.B; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

The calculation involved in the task, 32 - 3, is one of the simplest calculations in the Math Milestones tasks for grade 2.

out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:[‡]

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 2:6 is called "Put Together/ Take Apart with One Addend Unknown." It is a Put Together/Take Apart situation because cutting a rope 'takes it apart' into two pieces; and more specifically, the situation is "Put Together/Take Apart with One Addend Unknown" because the initially unknown quantity is the length of one of the two pieces of rope.

During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are extended into working with fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

The context in task 2:6 involves length measurement. Thus, the "objects" being counted in this problem are length units. This contrasts with problems involving more tangible objects such as apples or grapes. It is important to include length contexts in grade 2 word problems, because relating addition and subtraction to length, including by working with number lines, is part of early preparation for working with fractional quantities in grades 3 and later.

Task 2:6 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("29 feet"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between addition and subtraction.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document (p. 13) stresses that if textbooks and teachers model representations or solution methods, then "these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context."

Some equation models describe a situation in an algebraic way, such as ? + 3 = 32 (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 2:6? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 2:6? In what specific ways do they differ from 2:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 2:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ See Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

unknown number, such as ? = 32 - 3 (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

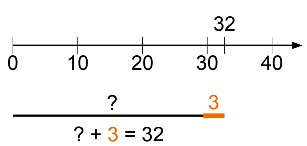
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: concepts of length measurement and length units; adding or subtracting a single-digit number to a two-digit number; working with the count sequence within 100; and writing and discussing situation equations and/or solution equations.

Extending the task

How might students drive the conversation further?

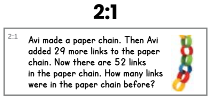
- For each number in an equation model for the situation, students could identify the quantity in the situation to which the number refers. For example, 32 refers to "the length of the rope before it was cut" or "the length of the two pieces if you put them together." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could create a schematic number line diagram representation of the situation and draw connections to the situation equation (see figure).



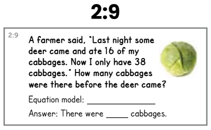
- Instead of cutting off a 3-foot piece of the rope, suppose we had cut the rope in half. How long would the pieces be in that case?

Related Math Milestones tasks

2:1


2:1 Avi made a paper chain. Then Avi added 29 more links to the paper chain. Now there are 52 links in the paper chain. How many links were in the paper chain before?

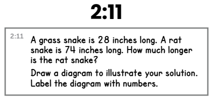
2:9


2:9 A farmer said, "Last night some deer came and ate 16 of my cabbages. Now I only have 38 cabbages." How many cabbages were there before the deer came?

Equation model: ______________

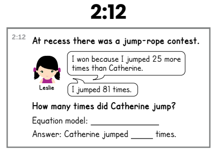
Answer: There were _____ cabbages.

2:11


2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?

Draw a diagram to illustrate your solution. Label the diagram with numbers.

2:12

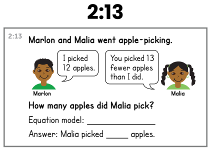

2:12 At recess there was a jump-rope contest.

How many times did Catherine jump?

Equation model: ______________

Answer: Catherine jumped _____ times.

2:13

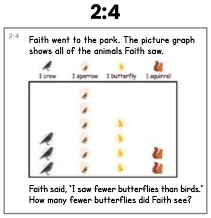

2:13 Marlon and Malia went apple-picking.

How many apples did Malia pick?

Equation model: ______________

Answer: Malia picked _____ apples.

2:4

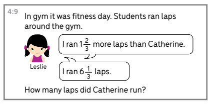

2:4 Faith went to the park. The picture graph shows all of the animals Faith saw.

Faith said, "I saw fewer butterflies than birds." How many fewer butterflies did Faith see?

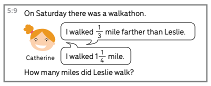
Other word problems and their situation types in grade 2 are as follows: tasks **2:1 Paper Chain**, *Add To with Start Unknown*; **2:9 Disappearing**

Cabbages, *Take From with Start Unknown*; **2:11 Grass Snake vs. Rat Snake**, *Compare with Difference Unknown ('how many more'/'how much longer' language)*, in a context involving length; **2:12 Jump-Rope Contest**, *Compare with Smaller Quantity Unknown ('more' language)*; **2:13 Apple-Picking**, *Compare with Bigger Quantity Unknown ('fewer' language)*; and **2:4 Animals in the Park**, combination of *Put Together/Take Apart with Total Unknown* and *Compare with Difference Unknown ('how many fewer' language)*.

4:9



5:9



In later grades, tasks **4:9 Fitness Day** and **5:9 Walkathon** are addition/subtraction word problems of situation type *Compare with Smaller Quantity Unknown*.

In earlier grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?