3:10 Alice's Multiplication Facts

Teacher Notes

Math Milestones™

Central math concepts

Parts (1) and (2). The number 7 can be decomposed as 5 + 2; that is, 5 + 2 is another way of writing 7. So, 7 × 8 = (5 + 2) × 8. The distributive property tells us that (5 + 2) × 8 = (5 × 8) + (2 × 8). This is useful if we remember that 8 × 5 = 40 and 8 × 2 = 16, because then we can say that 8 × 7 = 40 + 16. (As was done here, students can be encouraged to use parentheses to remind them that (5+2) is just a number, another way of writing 7.)

The distributive property initially emerges in the curriculum as a consequence of the equal-groups idea of multiplication. However, the distributive property remains applicable not only in grade 3 but also all throughout students' evolving ideas about multiplication, from grade 3 ideas of equal-groups to grades 4 and 5 ideas of times-as-much and scaling, to middle-grades ideas of scaling with signed rational numbers and real numbers, to high school ideas of complex number products. And from the middle grades onward, the distributive property will be the workhorse of algebra with variables and variable expressions. The distributive property is a sturdy principle that stretches across many years of school mathematics, available to be used whenever helpful to connect new ideas with old, to scaffold students' sense-making, to provide representations as a basis for creating and critiquing mathematical arguments, and to extend students' procedural knowledge and fluency with calculations and symbolic manipulations.

Part (3). A way to promote algebraic thinking in arithmetic is to help students view multiplication expressions like 5 × 8 as objects with structure that can be interpreted. An additional way to promote algebraic thinking in arithmetic is to help students to think about statements that refer to infinitely many cases. Students can wonder if what's true for particular numbers might be true for other numbers or all numbers. Part (3) of task 3:10 stretches beyond the particular case of 'the fact that Alice forgot' to aim at an important general principle. The thrust of the problem isn't just that 40 + 16 = 56, or that (5 × 8) + (2 × 8) = 7 × 8, but that there is a principle here that holds true for any numbers. Using letters to stand for 'any number,' this principle is $(b + c) \times a = b \times a + c \times a$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying by 2 and by 5; writing multiplication expressions to describe configurations of objects in equal groups; adding two two-digit numbers without regrouping; writing equations to express relationships; and basing multiplicative reasoning and distributive property reasoning on math diagrams.

3:10 Alice forgot what 7 × 8 equals. Alice knows that 5 × 8 = 40 and 2 × 8 = 16.

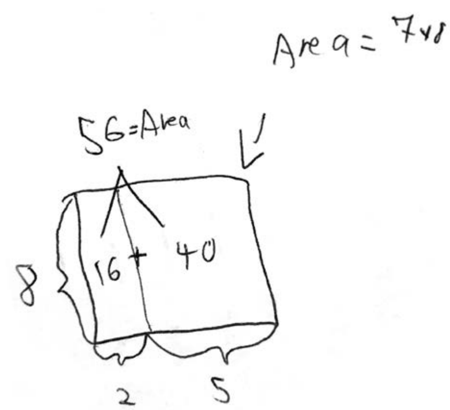
(1) Write a sentence to tell Alice how she can find the value of 7 × 8 by using the two facts she knows.

(2) Draw a diagram that could help Alice understand why your method works.

(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

Answer

(1) Answers will vary but should be based on the fact that 7 × 8 = 5 × 8 + 2 × 8. **(2)** Diagrams can vary but could use area, arrays, and/or equal groups to show the relationship between the products 2 × 8, 5 × 8, and 7 × 8 (see example). **(3)** Answers will vary but should involve the idea that products can always be broken into partial products, and/or the idea that a diagram like the student's diagram from part (2) could be drawn for any pair of factors.



Click here for a student-facing version of the task.

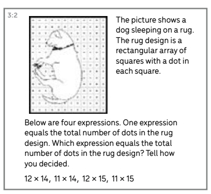
Extending the task

How might students drive the conversation further?

- Students who grasp the underlying distributive property principle at work in the task may still struggle to communicate the principle they understand. A routine such as *Stronger and Clearer Each Time* could be used to refine the explanations students give each other.
- Try all the decompositions of 7 into sums of 2 numbers: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1. For example, 7 × 8 = 1 × 8 + 6 × 8 = 8 + 48 = 56. Which decomposition makes the calculation of 7 × 8 easiest for you? Why are the results of all the decompositions the same?
- Students could extend the method to cases involving subtraction, such as finding the value of 4 × 8 by remembering 5 × 8 = 40 and subtracting 8 from 40.
- Students could use the biggest product they know to produce an even bigger product they didn't know before. For example, if a student knows 9 × 9 = 81, then they know that 9 9s are 81, so 18 9s must be 81 + 81; that is, 18 × 9 = 162. (Note that in grade 4, procedures for multiplying multi-digit numbers will use place-value-based decomposition 18 × 9 = 10 × 9 + 8 × 9 over arbitrary decompositions like 18 × 9 = 9 × 9 + 9 × 9, but both decompositions are valid applications of distributivity.)

Related Math Milestones tasks

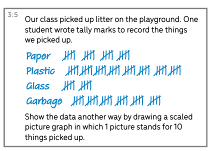
3:2


3:2 The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12 × 14, 11 × 14, 12 × 15, 11 × 15

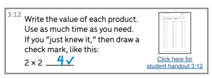
3:5


3:5 Our class picked up litter on the playground. One student wrote tally marks to record the things we picked up.

Paper
Plastic
Glass
Garbage

Show the data another way by drawing a scaled picture graph in which 1 picture stands for 10 things picked up.

3:12

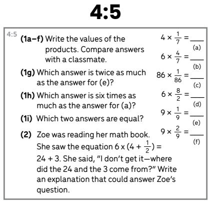

3:12 Write the value of each product. Use as much time as you need. If you "just knew it," then draw a check mark, like this:

2 × 2 4✓

Click here for student handout 3:12

Task **3:2 Hidden Rug Design** involves the equal-groups concept of multiplication and the practice of interpreting the structure of expressions. Task **3:5 Playground Cleanup** is a data representation task that involves equal groups of 5 and 10. Task **3:12 Products of Single-Digit Numbers** is a fluency and recall task for these products.

4:5


4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

(1g) Which answer is twice as much as the answer for (e)?

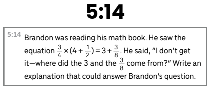
(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)

5:14


5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

In later grades, task **4:5 Fraction Products and Properties** (part (2)) involves the distributive property as applied to a product of a whole number and a fraction; task **5:14 Brandon's Equation** follows that progression further, featuring a product in which neither factor is a whole number.

Refer to the Standards

3.OA.B.5, 3.MD.C.7b; MP.1, MP.3, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

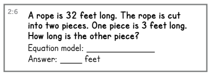
Additional notes on the design of the task

The products 5 × 8 and 2 × 8 are typically easier for students to remember than the product 7 × 8, which makes the strategy of this task pragmatically useful in addition to mathematically important. Calculating an unknown product using known partial products is an important strategy for students during the time when they are still on the path to becoming fluent with and remembering single-digit products. This strategy will also underlie multi-digit multiplication algorithms in later grades (see for example task **4:10 Calculating Products and Quotients**).

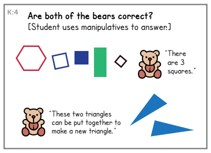
Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:10? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 3:10? In what specific ways do they differ from 3:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

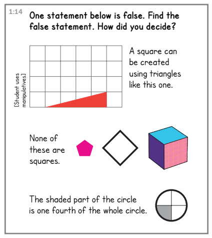
2:6



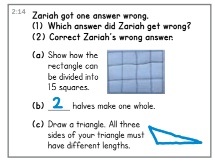
K:4



1:14



2:14



In earlier grades, task **2:6 Cutting a Rope** is a word problem whose equation model consists of a decomposition, 32 = 29 + 3. Spatially, tasks **K:4 Bears Talk About Shapes**, **1:14 Shape True/False**, and **2:14 Correcting a Shape Answer** involve composing and/or decomposing shapes.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?