3:11 Water Balloons

Teacher Notes

Math Milestones™

Central math concepts

Table 2 of the *Progression* document† lists elementary addition and subtraction situations (p. 9). Table 3 of the document lists elementary multiplication and division situations (p. 23). These elementary situations combine in multi-step problems. Task 3:11 combines an addition situation, *Put Together/Take Apart with Total Unknown*, and a multiplication/division situation, *Equal Groups of Objects with Group Size Unknown*.

Equations are an important way to represent the relationships between quantities in a situation. In task 3:11, the equations

$24 + 24 = 48$

$48 \div 6 = 8$

could be seen as "telling the story" of the problem in the following way. First, the equation $24 + 24 = 48$ describes a process of Steven and Hawa putting their balloons together. Then, the equation $48 \div 6 = 8$ describes a process of 6 friends sharing those balloons equally.

Another way to tell the story could be with the following equations:

$24 \div 6 = 4$

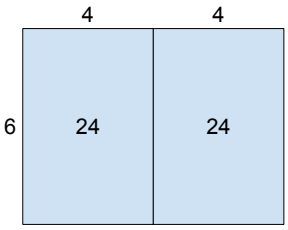
$24 \div 6 = 4$

$4 + 4 = 8.$

In this version of the story, one of the friends first receives a share of Steven's balloons (as represented in the first equation $24 \div 6 = 4$) and next receives a share of Hawa's balloons (as represented in the second equation $24 \div 6 = 4$). Then the equation $4 + 4 = 8$ shows those two shares being combined.

It is intuitively clear that the situation on the playground could have played out in either way, but how does it happen mathematically that the final answer comes out the same regardless of which of these two approaches we choose? The guarantee in this case is the distributive property:

$(24 + 24) \div 6 = 24 \div 6 + 24 \div 6.$



Relating an equation to the situation it describes involves viewing an expression like $24 + 24$ not only as "a step" toward a result of 48, but also as an object that can be interpreted before evaluating it, based on the meaning of the operation and the role of the numbers in the situation. $24 + 24$ equals 48, to be sure, but $24 + 24$ left unevaluated is also a valuable record of an operation of *putting two collections together*. This

3:11 Steven, Hawa, and 4 more friends went to the park. Steven brought 24 water balloons. Hawa brought 24 water balloons. Nobody else brought water balloons. The 6 friends shared all the water balloons equally. How many water balloons did each friend get?

Answer

Each friend got 8 water balloons.

Click here for a student-facing version of the task.

Refer to the Standards

3.OA.D.8; MP.1, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

The number 4 in the first sentence likely doesn't enter into the solution of the problem, unless it is used to determine the total number of friends. This value is stated explicitly later in the text of the problem.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:11? In what specific ways do they differ from 3:11?

perspective on a numerical expression expands students' perspective on arithmetic, prefiguring their work with algebraic expressions in the middle grades (CCSS 6.EE.A.2). As observed in the *Progression* document,

> [Students] understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two two-digit numbers; using addition in context; finding single-digit products and related quotients; using multiplication and division in context; and writing situation equations and solution equations.

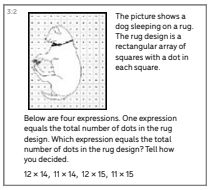
Extending the task

How might students drive the conversation further?

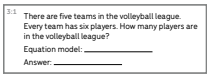
- Students could be asked how many water balloons each friend would get if a third friend had brought 6 water balloons to add to the total.
- For that version of the problem, students could compare and discuss two methods, such as (a) 48 + 6 = 54, 54 ÷ 6 = 9; (b) 8 + 1 = 9 because 6 more balloons provides 1 more balloon for each friend.

Related Math Milestones tasks

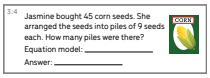
3:2


3:2
The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.
Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.
12 × 14, 11 × 14, 12 × 15, 11 × 15

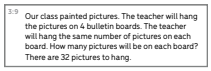
3:1


3:1
There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?
Equation model: _____
Answer: _____

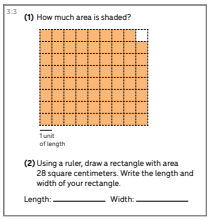
3:4


3:4
Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?
Equation model: _____
Answer: _____

3:9


3:9
Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:3


3:3
(1) How much area is shaded?
1 unit of length
(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.
Length: _____ Width: _____

Task **3:2 Hidden Rug Design** centers on the equal-groups concept of multiplication in an array context, in a way that involves viewing expressions as objects with structure. Tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, and **3:9 Bulletin Board Pictures** are word problems centered on equal-groups concepts of multiplication and/or division. Multiplication is useful in task **3:3 Length and Area Quantities**.

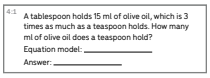
Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft) K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

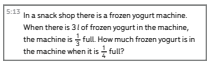
* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

4:1



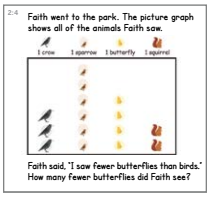
4:12

5:13



In later grades, tasks **4:1 A Tablespoon of Oil** and **4:12 Super Hauler Truck** are word problems involving multiplicative comparison (see Table 3, p. 23 of the relevant *Progression* document). Task **5:13 Frozen Yogurt Machine** combines an unknown-factor situation with an unknown-product situation, in a context involving fractional quantities.

2:4



In earlier grades, task **2:4 Animals in the Park** combines the situation types *Put Together/Take Apart with Total Unknown and Compare with Difference Unknown ('how many fewer' language)* (see Table 2, p. 9 of the relevant *Progression* document).

3:11 Water Balloons

Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?