3:12 Products of Single-Digit Numbers

Teacher Notes

Math Milestones™

Central math concepts

Task 3:12 draws on memory and fluency. The table illustrates how the 66 brief problems in task 3:12 map to the multiplication table.

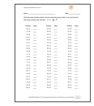
×	0	1	2	3	4	5	6	7	8	9
0										
1		✔						✔		
2				✔	✔	✔	✔	✔	✔	✔
3			✔	✔	✔	✔	✔	✔	✔	✔
4		✔	✔	✔	✔	✔	✔	✔	✔	✔
5			✔	✔	✔	✔	✔	✔	✔	✔
6			✔	✔	✔	✔	✔	✔	✔	✔
7			✔	✔	✔	✔	✔	✔	✔	✔
8			✔	✔	✔	✔	✔	✔	✔	✔
9			✔	✔	✔	✔	✔	✔	✔	✔

The values of the single-digit products play a large role in school mathematics during the elementary grades, the middle grades, and high school. For example, the problem of calculating a multi-digit product can be reduced to summing a series of terms that are single-digit products multiplied by powers of ten (see sidebar). The standard pencil-and-paper algorithm for multi-digit multiplication is an efficient bookkeeping method for this process.

> **Rewriting the product 734 × 8 as a sum of single-digit products multiplied by powers of ten:**
>
> $734 \times 8 = (700 + 30 + 4) \times 8$
>
> $= 700 \times 8 + 30 \times 8 + 4 \times 8$
>
> $= (\mathbf{7 \times 8}) \times 100 + (\mathbf{3 \times 8}) \times 10 + (\mathbf{4 \times 8})$.

Indeed, many mathematical tasks in grade 3 and beyond are facilitated by remembering single-digit products and being fluent with related quotients. Examples include:

- Finding final answers to word problems in multiplication and division situations;
- Calculating multi-digit products and quotients, and assessing the results of such calculations by estimating;
- Understanding, recognizing, and generating equivalent fractions;
- Reasoning about products and quotients of fractions;
- Factoring composite numbers and multiplying out prime factorizations;
- Understanding, recognizing, and generating equivalent ratios, and seeing patterns in ratio tables;
- Reasoning with ratios and seeing patterns in ratio tables;

> 3:12 Write the value of each product. Use as much time as you need. If you "just knew it," then draw a check mark, like this:
>
> 2 × 2 4✓
>
> 
>
> Click here for student handout 3:12

Answer

Click here for an answer key. Students' check marks may vary by individual.

Click here for a student-facing version of the task.

Refer to the Standards

3.OA.C.7; MP.1, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

- The task is designed to be worked on after all the products in the multiplication table have been understood and practiced.
- The instructions say, "Use as much time as you need." Reasons for this include: (1) Differentiating between students on the basis of their speed isn't the purpose of the task. (2) More generally, speed isn't an important disciplinary value in mathematics. (3) Emphasizing speed in the mathematical community of the classroom can have negative effects on students' mathematics identity.

- Solving problems involving unit rates, linear functions, percents, unit conversions, similar figures, and other instances of scaling and proportionality;
- Factoring quadratic expressions and other polynomial expressions; and
- Counting possible outcomes to determine probabilities.

Remembering single-digit products and being fluent with related quotients is therefore an important goal (CCSS 3.OA.C.7). This goal needs to be reached by an intellectually valid, emotionally supportive learning path. The mathematical stages of that path are articulated in the *Progression* document, under the heading "Levels in problem representation and solution" (see pp. 25–27).†

As noted also in the *Progression* document (p. 27), "Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible. Because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two one-digit numbers. … [T]his isn't a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning."‡ Also note (p. 22) that "mastering this material, and reaching fluency in single-digit multiplications and related divisions with understanding, may be quite time consuming …. So it is imperative that extra time and support be provided if needed." In addition, grades 3 and 4 teachers could co-develop a plan for extending and/or maintaining recall and fluency with single-digit products and related quotients, as needed, during grade 4.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: Level 3 strategies as described in the *Progression* document under the heading "Levels in problem representation and solution" (see pp. 25–27).

Extending the task

How might students drive the conversation further?

- Students could notice that a certain factor is absent from the student handout, or be asked which factor is absent. What might be the reason why no products with this factor were included?
- Students could circle several products they feel they could use more practice with.

Additional notes on the design of the task (continued)

- The instructions say, "If you 'just knew it,' then draw a check mark." This is intended to provide information about which single-digit products are known from memory.
- The task includes relatively few products that involve 1 as a factor, and it includes no products that involve 0 as a factor. All such products are instances of the general patterns $1 \times n = n \times 1 = 1$ and $0 \times n = n \times 0 = 0$. (See "Extending the task.")

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:12? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 3:12? In what specific ways do they differ from 3:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft) K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ For further discussion of such an interwoven process, see "By the end of grade 3: Developing fluency with multiplication and division" (blog post by Hill, 2021).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Related Math Milestones tasks

3:13

3:13 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 3:13

3:1

3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: ________

Answer: ________

3:4

3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: ________

Answer: ________

3:9

3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:11

3:11 Steven, Hawa, and 4 more friends went to the park. Steven brought 24 water balloons. Hawa brought 24 water balloons. Nobody else brought water balloons. The 6 friends shared all the water balloons equally. How many water balloons did each friend get?

3:10

3:10 Alice forgot what 7 × 8 equals. Alice knows that 5 × 8 = 40 and 2 × 8 = 16.

(1) Write a sentence to tell Alice how she can find the value of 7 × 8 by using the two facts she knows.

(2) Draw a diagram that could help Alice understand why your method works.

(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

Task **3:13 Fluency within the Multiplication Table** includes problems about fact families, such as 21 ÷ 7 = ☐, ☐ × 8 = 16, ☐ ÷ 3 = 5, and 12 ÷ ☐ = 2, in which an unknown number is sought that makes an equation true. Single-digit products and related quotients are involved in the word problems in tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, **3:9 Bulletin Board Pictures**, and **3:11 Water Balloons**. Task **3:10 Alice's Multiplication Fact** involves a distributive property strategy for using known products to determine an unknown product.

4:10

4:10 Write the values of the products and quotients. Check the quotients by multiplying.

Mentally: 40 × 20, 30 × 11, 12 × 60, 5 × 19, 480 ÷ 8

With pencil and paper: 6,132 × 6; 48 × 39; 7)8,722

5:5

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10}$ ÷ 10 = ?	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3}$ = ?	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8}$ = ?	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5}$ = ?	0.72 − 0.17 = ?
	$\frac{1}{3}$ ÷ (6 × 5) = ?	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

6:14

6:14 *Pencil and paper* **(1)** 81.53 ÷ 3.1 = ? **(2)** $\frac{7}{8} \div \frac{2}{3}$ = ? **(3)** Check both of your answers by multiplying.

In later grades, task **4:10 Calculating Products and Quotients** involves grade-level procedures with multi-digit multiplication and division. Task **5:5 Calculating** continues procedures into larger numbers of digits and into fractions and decimals. Task **6:14 Dividing Decimals and Fractions** marks the culmination of algorithmic procedures with general fraction and decimal division.

2:5

2:5 Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2 4✓

Click here for student handout 2:5

In earlier grades, task **2:5 Sums of Single-Digit Numbers** is the analogue of task 3:12 for addition.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?