3:2 Hidden Rug Design

Teacher Notes

Central math concepts

Calculating the value of a product of two two-digit numbers isn't part of the expected learning in grade 3. However, even if one doesn't know the value of 12 × 15, one can still represent the total number of dots in the rug design by the expression 12 × 15. That's possible thanks to the fundamental idea behind early multiplication work, which is that for two whole numbers *A* and *B*, the expression *A* × *B* means the total number of objects in *A* groups of *B* objects each. Applying this idea to the rug design, if we think of each row as a group, then there are 15 groups with 12 objects in each group; equivalently, if we think of each column as a group, then there are 12 groups with 15 objects in each group. Taking the second point of view, the number of dots must be equal to 12 x 15—even if we don't know what number that is!

There's an almost magical power in being able to write an expression for the total number of dots without being able to see all of the dots. Wielding that power requires some willingness on the part of the student to view an expression like 12 × 15 not just as a calculation problem ('What is the value of 12 × 15?') but also as a representation problem that involves recognizing the uses and meanings of multiplication. Each factor in the expression 12 × 15 can be connected to particular features of the context at hand, and the expression as whole can also be connected to the context. This is one important way in which elementary-grades students "look for and make use of structure" (CCSS MP.7).

Relevant prior knowledge

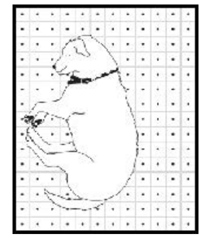
The following mathematics knowledge may be activated, extended, and deepened while students work on the task: spatial structuring and partitioning; rectangular array structures; and fundamental concepts of multiplication with whole numbers.

Extending the task

How might students drive the conversation further?

- Students could choose to satisfy their curiosity by determining the numerical value of 12 × 15 using skip-counting methods or the distributive property. A direct distributive property approach might begin with reasoning that 12 × 15 is ten 15s plus two more 15's, then proceed from there. An implicit distributive property approach might be to break 12 × 15 into a sum, for example 4 × 15 + 4 × 15 + 4 × 15, then work to determine the value of 4 × 15.
- Students could consider a more intricate rug design in which there are 3 dots in each square. Given this design, what is a multiplication expression for the total number of dots?

3:2



The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12 × 14, 11 × 14, 12 × 15, 11 × 15

Answer

12 × 15. Explanations may vary but should involve the idea that 12 × 15 means the total number of objects in 12 groups of 15 objects each or, equivalently, the total number of objects in 15 groups of 12 objects each. (The number of groups and the number of objects in each group depend on whether we view the array as a collection of columns or a collection of rows.)

Click here for a student-facing version of the task.

Refer to the Standards

3.OA.A.1; MP.2, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Related Math Milestones tasks

3:1

3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: ____________

Answer: ____________

3:4

3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: ____________

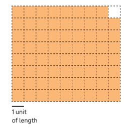
Answer: ____________

3:9

3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:3

3:3 **(1)** How much area is shaded?



(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ________ Width: ________

Tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, and **3:9 Bulletin Board Pictures** form a kind of survey of the essential early meanings of multiplication and division; the requested equation models can support learning about the relationship between the operations. Multiplication is useful in task **3:3 Length and Area Quantities**.

4:1

4:1 A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

Equation model: ____________

Answer: ____________

4:12

4:12 The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?

4:10

4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

In later grades, tasks **4:1 A Tablespoon of Oil** and **4:12 Super Hauler Truck** involve an important extension of the multiplication concept into times-as-much thinking and multiplicative comparisons. Task **4:10 Calculating Products and Quotients** involves calculating products of two two-digit numbers.

2:2

2:2 **(1) True or false?**

(a) 2 hundreds + 3 ones > 5 tens + 9 ones

(b) 9 tens + 2 hundreds + 4 ones < 924

(c) 456 < 5 hundreds

(2) Write the number that makes each statement true.

(a) 7 ones + 5 hundreds = _____

(b) 14 tens = _____

(c) 90 + 300 + 4 = _____

In earlier grades, part (1) of task **2:2 Place Value to Hundreds** could be answered by comparing parts of the expressions on either side of the inequality symbol (instead of calculating the values of those expressions).

Additional notes on the design of the task

- It is not the intent that students be able to calculate or estimate the numerical value of the listed products. It is also not the intent that students try to count the dots that are visible, or that students estimate how many dots are hidden by the dog. If students carry out these operations, the results can be gathered as partial knowledge about the situation.
- The importance of the language “rectangular array” in the task is that it guarantees that there is a regular pattern to the dots and their placement, so that students can draw a valid inference about the total number of dots even though some of the dots are hidden by the dog.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:2? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:2? In what specific ways do they differ from 3:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?