3:3 Length and Area Quantities

Teacher Notes

Central math concepts

In grades K–2, students work with length units and concepts of length measurement (CCSS 1.MD.A.2). Initially in kindergarten, students make non-numerical comparisons of length and other measurable quantities (CCSS K.MD.A). Then in grade 1, using objects as length units, students learn that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. In grade 2, students extend these ideas to abstract length units and relate length measurement to addition, subtraction, and the number line (CCSS 2.MD.A).

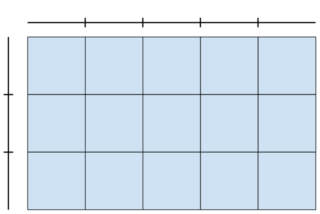
After studying length as a measurable quantity in the primary grades, students learn to recognize area as a measurable attribute of plane figures. Area is the amount of two-dimensional surface a figure contains.

Consistent with this idea, congruent figures are assumed to enclose equal areas. Students also understand the concepts involved in measuring area (CCSS 3.MD.C.5):

- **A unit of measure for area:** An area unit is built from a chosen length unit. Given a length unit, a square with side length equal to 1 unit, called "a unit square," is said to have "one square unit" of area.
- **Quantifying area:** A plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

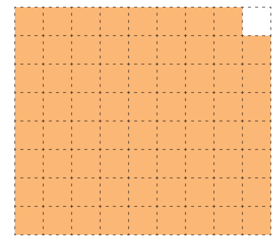
For example, in task 3:3, one perceives that the quantity of shaded area is less, by a single square unit, than it would have been had the entire outer dotted rectangle been shaded.

Area units and length units are closely connected. To see this, imagine tiling a rectangle with unit squares, as shown in the figure. This creates a rectangular array of squares. The number of squares in the array can be found by multiplying the number of squares in each row by the number of rows. An important observation about this figure is that the number of squares in each row of the array equals the number of length units in one of the sides, while the number of rows equals the number of length units in one of the adjacent sides. Thus, the number of squares in the array can be found by multiplying the number of length units in one side by the number of length units in an adjacent side. This conclusion is summarized by the rectangle area formula, $A = L \times W$. In this formula:



- L is the length of one side of the rectangle, measured in length units.
- W is the length of an adjacent side of the rectangle, measured in the same length units that W is measured in.
- A is the number of unit squares needed to tile the rectangle, where a unit square has sides of length equal to 1 length unit (the same length unit that L and W were measured in).

3:3 **(1)** How much area is shaded?

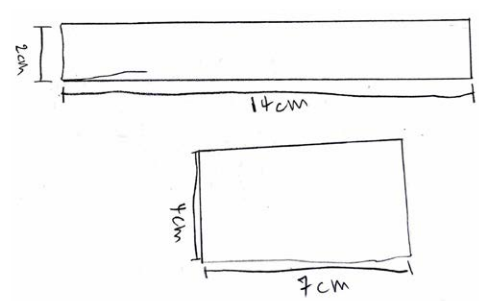


(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ________ Width: ________

Answer

(1) 71 square units. **(2)** See examples. Rectangles that are slightly irregular-looking can still be accurate enough to be considered correct. Rectangles may be rotated at any angle relative to the examples shown. Which side is identified as the length and which side is identified as the width is unimportant, unless these terms have been defined unambiguously in the classroom discourse, in which case the terms should be used as defined. Rectangles such as $3\frac{1}{2}$ cm by 8 cm with one or more fractional side lengths are not expected, but no rectangle is incorrect if its area is 28 square centimeters.



Click here for a student-facing version of the task.

As the above discussion shows, understanding area measurement requires being able to see a blank rectangular region as decomposable into rows and columns of squares. This skill is called spatial structuring (see the Teacher Notes for task **2:14 Correcting a Shape Answer**). In part (1) of task 3:3, the dashed lines provide the spatial structuring.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: spatially structuring a rectangle into an array; decomposing 28 as a product of two factors or as a sum of equal addends; and working with measurement concepts.

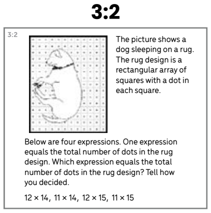
Extending the task

How might students drive the conversation further?

- If some students draw a 4-by-7 rectangle for part (2), while other students draw a 2-by-14 rectangle, students could use grid paper to cut up a 2-by-14 rectangle and show that its parts can exactly cover a 4-by-7 rectangle.
- If students did not show unit squares in their answer for part (2), they could be asked to draw in the unit squares and to verify that there are 28 of them.
- In part (1), if a student found the answer as 9 × 8 - 1, then since 9 × 8 = 8 × 8 + 8, the answer could be expressed in a different way as 8 × 8 + 8 - 1, which is 8 × 8 + 7. Is there a way to connect the expression 8 × 8 + 7 to the shape of the shaded region?

Related Math Milestones tasks

3:2

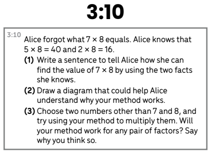

3:2

The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12 × 14, 11 × 14, 12 × 15, 11 × 15

3:10


3:10

Alice forgot what 7 × 8 equals. Alice knows that 5 × 8 = 40 and 2 × 8 = 16.

(1) Write a sentence to tell Alice how she can find the value of 7 × 8 by using the two facts she knows.

(2) Draw a diagram that could help Alice understand why your method works.

(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

Task **3:2 Hidden Rug Design** relates a product expression to the number of dots in a rectangular array. In part (2) of task **3:10 Alice's Multiplication Fact**, an area diagram could be used to illustrate a distributive property relationship.

Refer to the Standards

3.MD.C; MP.2, MP.5, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

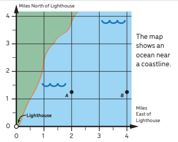
- The statement of part (1) of the task doesn't refer explicitly to a unit of area; this is intended to help reveal the way in which the student talks about or uses the concept of an area unit, when the phrasing of a task doesn't scaffold that use.
- The intent of the relatively large dimensions of the outer dotted rectangle is to establish a situation in which multiplying is more valuable than counting-all.
- Part (2) of the task relates to multiplication in that it involves determining, say, that 28 = 4 × 7 or that 28 = 14 + 14 = 2 × 14. This element of part (2) is less important, however, than the contemplative act of drawing the rectangle, which creates space for relating the familiar idea of a length unit to the newer idea of an area unit, and which also invites thinking about geometric relationships in a rectangle.
- The drawing in part (2) is preferably made on blank paper, not lined paper or grid paper. This ensures that the lines and grid lines do not clash with the student's plan, and it allows the possibility for the student to do the spatial structuring by drawing lines within the rectangle.

4:13

4:13 **(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.
(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?
(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

5:7

5:7

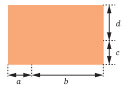


The map shows an ocean near a coastline.

Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark *C* and *D* on the map and shade rectangle *ABCD*. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

6:11

6:11 The diagram shows a rectangle. The variables *a*, *b*, *c*, and *d* are lengths in meters.



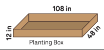
(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

6:12

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

5:3

5:3 A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is $54\ ft^3$ of soil. How many truckloads of soil will fill all of the boxes?



Planting Box

8:12

8:12 Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled ($1\ m^3$ of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

In later grades, task **4:13 Area Units** involves a quantity of area measured using two different area units. Task **5:7 Shipwrecks** involves rectangle area in context for a rectangle with fractional dimensions, and task **6:11 Area Expressions** involves rectangle area in a case where the lengths are variables rather than numbers. Task **6:12 Coordinate Triangle** involves area measure for a triangle. Tasks **5:3 Neighborhood Garden** and **8:12 Fish Tank Design** are applications involving volume measurement.

1:3

1:3 Using a paper clip as a unit of length, draw a straight line 7 units long.

2:14

2:14 Zariah got one answer wrong.
(1) Which answer did Zariah get wrong?
(2) Correct Zariah's wrong answer.

(a) Show how the rectangle can be divided into 15 squares.

(b) 2 halves make one whole.

(c) Draw a triangle. All three sides of your triangle must have different lengths.

In earlier grades, task **1:3 Paper Clip Length Units** involves creating a length from units. Task **2:14 Correcting a Shape Answer** (part (2)(a)) involves spatial structuring.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:3? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:3? In what specific ways do they differ from 3:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?