3:4 Corn Seeds

Teacher Notes

Central math concepts

Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations, so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

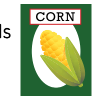
Problems involving division also implicitly involve multiplication, because division finds an unknown factor. That is, $C \div A$ is the unknown factor in $A \times \square = C$. This is why a division calculation is checked by multiplying. From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between multiplication and division can play out in situations in conceptually distinct ways, including:[‡]

- **Equal Groups:** Product Unknown, Group Size Unknown, and Number of Groups Unknown

3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: ____________

Answer: ____________



Answer

Equation model: $\square \times 9 = 45$, $45 \div 9 = \square$, or an equivalent equation with a symbol for the unknown number, or an equivalent solved equation, such as $5 \times 9 = 45$. Answer: There were 5 piles of seeds.

Click here for a student-facing version of the task.

Refer to the Standards

3.OA.A.3, 3.OA.A; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

The reason the seeds in the task are corn seeds is that many other kinds of garden seeds (such as lettuce seeds) are very small, and those kinds of seeds would be difficult to handle one by one.

- **Arrays:** Product Unknown, Number of Rows Unknown, and Number of Columns Unknown
- **Compare:** Size of Larger Quantity Unknown, Size of Smaller Quantity Unknown, and Multiplier Unknown

In particular, the situation type in task 3:4 is called "Equal Groups with Number of Groups Unknown." It is an Equal Groups situation because each pile contains the same number of seeds. And the situation is "Equal Groups with Number of Groups Unknown" because the initially unknown quantity is the number of piles of seeds.

Students might determine the value of the unknown factor in □ × 9 = 45 by remembering that 5 × 9 = 45, or they might calculate the quotient 45 ÷ 9 = 5 by various methods. In the *Progression* document (pp. 25–27), three levels of representation and solution are articulated for multiplying and dividing within the times tables:§

- **Level 1** is making and counting all of the quantities involved in a multiplication or division. Representing the quantities with a diagram affords reflection and sharing when it is drawn on the board and explained by a student.
- **Level 2** is repeated counting on by a given number, such as for 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 × 3, you know the number of 3s and count by 3 until you reach 8 of them. For 24 ÷ 3, you count by 3 until you hear 24, then look at your tracking method to see how many 3s you have. Because listening for 24 is easier than monitoring the tracking method for 8 3s to stop at 8, dividing can be easier than multiplying.
- **Level 3** is using the commutative, associative, and/or distributive properties to replace the given problem with an easier problem or subproblems. For example,
  - 4 × 6 can be replaced with 8 × 3:

    $$4 \times 6 = 4 \times (2 \times 3) = (4 \times 2) \times 3 = 8 \times 3.$$

  - Students may know a product 1 or 2 ahead of or behind a given product and say:

    I know 6 × 5 is 30, so 7 × 5 is 30 + 5 more, which is 35.

    This implicitly uses the distributive property:

    $$7 \times 5 = (6 + 1) \times 5 = 6 \times 5 + 1 \times 5 = 30 + 5 = 35.$$

  - Students may implicitly use the distributive property to decompose a product that they do not know in terms of two products they know (see the Teacher Notes for task **3:10 Alice's Multiplication Fact**). Students may not use the properties explicitly, but classroom discussion can identify and record properties in student reasoning. An area diagram can support such reasoning.

As applied to the quotient 45 ÷ 9 in task 3:4, a Level 3 (property-based) approach might be that if we know 3 9s are 27, then 27 is less than 45 by an amount 3 + 10 + 5 = 18, and that's 2 9s, so 45 is 5 9s. This approach implicitly uses the distributive property as follows:

$$27 + 18 = 45$$

$$3 \times 9 + 2 \times 9 = 45$$

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:4? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:4? In what specific ways do they differ from 3:4?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ See Table 3, p. 23 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

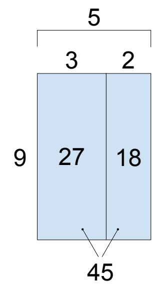
§ These descriptions of the levels quote from the *Progression* document, with some edits; follow the link for the full *Progression* text.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

$(3 + 2) \times 9 = 45$

$5 \times 9 = 45.$

The area diagram illustrates aspects of this calculation.



Another approach might be to think that since $90 \div 9 = 10$, then $45 \div 9 = 10 \div 2$:

$$\begin{aligned} 45 \div 9 &= 2 \times (45 \div 9) \div 2 \\ &= (90 \div 9) \div 2 \\ &= 10 \div 2 \\ &= 5. \end{aligned}$$

Task 3:4 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("5 piles of seeds"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between multiplication and division.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document stresses that "what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context" (p. 13).

Some equation models describe a situation in an algebraic way, such as $\square \times 9 = 45$, (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as $45 \div 9 = \square$ (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: Level 2 and 3 strategies for calculating products and quotients; finding quotients by remembering products; writing equations to describe quantitative relationships; and relating representations such as equations and diagrams to a situation.

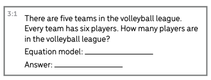
Extending the task

How might students drive the conversation further?

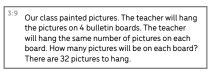
- For each number in an equation model for the situation, such as ☐ × 9 = 45, students could identify the quantity in the situation to which the number refers. For example, 45 refers to "the total number of seeds." Note that naming a quantity is different from naming the numerical value of the quantity.
- Students could check their quotient by multiplying.

Related Math Milestones tasks

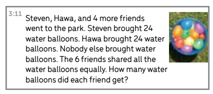
3:1


3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?
Equation model: ____________
Answer: ____________

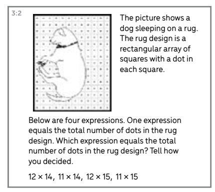
3:9


3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:11


3:11 Steven, Hawa, and 4 more friends went to the park. Steven brought 24 water balloons. Hawa brought 24 water balloons. Nobody else brought water balloons. The 6 friends shared all the water balloons equally. How many water balloons did each friend get?

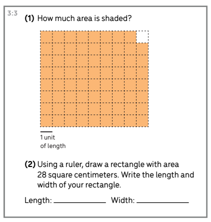
3:2


3:2 The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12 × 14, 11 × 14, 12 × 15, 11 × 15

3:3


3:3 (1) How much area is shaded?

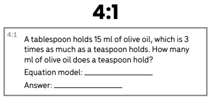
1 unit of length

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

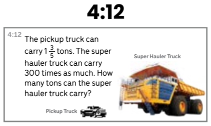
Length: ____________ Width: ____________

Task **3:1 Volleyball Players** is a word problem of type *Equal Groups with Product Unknown*; task **3:9 Bulletin Board Pictures** is a word problem of type *Equal Groups with Group Size Unknown*; and task **3:11 Water Balloons** is a multi-step word problem involving two situation types, one of which is *Equal Groups with Group Size Unknown*. Task **3:2 Hidden Rug Design** involves interpreting a product expression in an array context. Multiplication is useful in task **3:3 Length and Area Quantities**.

4:1


4:1 A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?
Equation model: ____________
Answer: ____________

4:12


4:12 The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?

In later grades, task **4:1 A Tablespoon of Oil** is a word problem of type *Compare with Size of Smaller Quantity Unknown*, and task **4:12 Super Hauler Truck** is a word problem of type *Compare with Size of Larger Quantity Unknown*.

In earlier grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

