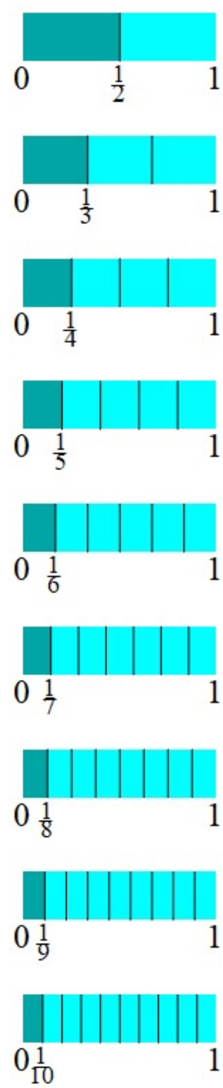
3:6 Unit Fraction Ideas

Teacher Notes

Central math concepts

Possibly the two most important foundations for fraction arithmetic are the idea of the unit fraction and the principle of fraction equivalence. Students will frequently employ unit fraction ideas and the principle of fraction equivalence throughout the upper-elementary grades as they learn to extend arithmetic from whole numbers to fractions. Task 3:6 invests in the first of these essential concepts, the unit fraction.



Unit fractions play an important role in school mathematics for at least three reasons. One is that everyday life commonly presents us with situations in which a measurement unit is too large to serve our purposes. For example, when hanging pictures on a wall, measuring to the nearest whole inch might not be precise enough, so we may choose to work with smaller units such as fourths or eighths. A second reason for the importance of unit fractions is that unit fractions allow students to apply unit thinking to fraction problems. As a grade 6 example, the quotient $\frac{6}{7} \div \frac{2}{7}$ can be seen to equal 3 for the same reason that 6 of any unit divided by 2 of that same unit equals 3. In this case, the units are *sevenths*.

A third reason why unit fractions are so important in school mathematics is the way they unify multiplication and division. Think for example about the idea that $\frac{4}{5}$ is the quantity formed by 4 parts when a whole is partitioned into 5 equal parts. That sounds like multiplication (because there are 4 of something), but it also sounds like division (because we are partitioning). By the upper elementary grades, fractions will be breaking down conceptual walls between multiplication and division. For example, dividing by B can be accomplished just as well by multiplying by $\frac{1}{B}$.†

When talking about fractions, it is desirable to talk as much as possible about *fractions*, not about whole numbers. One way to do this is to refer frequently to the units of thirds, fourths, fifths, etc., when possible, instead of referring only to denominators 3, 4, 5, etc. "Thirds are smaller than halves" is an important piece of quantitative thinking. A mnemonic device such as "To compare unit fractions, compare the denominators" contains much less mathematics and could be easy to misapply. Nevertheless, a general understanding of how the size of a unit fraction depends on the denominator is a valuable insight and a kind of early functional thinking.

3:6 Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

Answer

$\frac{1}{41}$ is greater than $\frac{1}{73}$. Explanations may vary. One kind of explanation involves the idea that if you partition a whole into a greater number of equal parts, then the parts have smaller size than when you partition a whole into a lesser number of equal parts. This might be illustrated with contexts, such as saying that one would rather receive $\frac{1}{41}$ of a bar of gold than $\frac{1}{73}$ of the bar of gold, because when you slice a bar of gold into 41 equal parts, those parts are larger (heavier) than when you slice the bar into 73 equal parts.

Click here for a student-facing version of the task.

Refer to the Standards

3.NF.A; MP.1, MP.5, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: spatial structuring and partitioning; unit ideas in measurement; and basing reasoning on math diagrams.

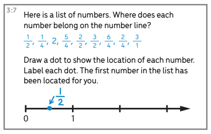
Extending the task

How might students drive the conversation further?

- Students could invent even "sillier" versions of the problem, such as the problem of comparing $\frac{1}{1{,}000{,}000}$ to $\frac{1}{1{,}000{,}001}$.
- Students could propose a mathematical discovery, for example by realizing that there is no smallest number. One justification for this is that a whole can be partitioned into any number of parts no matter how many.

Related Math Milestones tasks

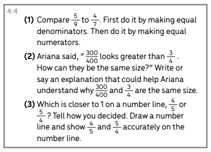
3:7


3:7 Here is a list of numbers. Where does each number belong on the number line?

$\frac{1}{2}, \frac{1}{4}, 2, \frac{5}{4}, \frac{2}{2}, \frac{3}{2}, \frac{6}{4}, \frac{2}{4}, \frac{3}{1}$

Draw a dot to show the location of each number. Label each dot. The first number in the list has been located for you.

4:4

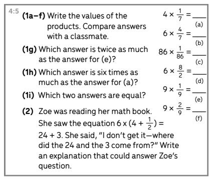

4:4

(1) Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.

(2) Ariana said, "$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?" Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.

(3) Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

4:5


4:5

(1a–f) Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ____ (a)

$6 \times \frac{4}{7} =$ ____ (b)

$86 \times \frac{1}{86} =$ ____ (c)

$6 \times \frac{8}{2} =$ ____ (d)

$9 \times \frac{1}{9} =$ ____ (e)

$9 \times \frac{2}{9} =$ ____ (f)

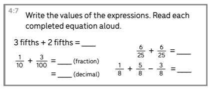
(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

4:7


4:7 Write the values of the expressions. Read each completed equation aloud.

3 fifths + 2 fifths = ____

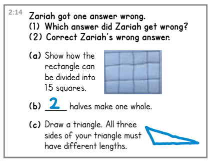
$\frac{1}{10} + \frac{3}{100} =$ ____ (fraction)

= ____ (decimal)

$\frac{6}{25} + \frac{6}{25} =$ ____

$\frac{1}{8} + \frac{5}{8} - \frac{3}{8} =$ ____

2:14


2:14 Zariah got one answer wrong.

(1) Which answer did Zariah get wrong?

(2) Correct Zariah's wrong answer.

(a) Show how the rectangle can be divided into 15 squares.

(b) 2 halves make one whole.

(c) Draw a triangle. All three sides of your triangle must have different lengths.

Task **3:7 Locating Numbers on a Number Line** integrates concepts of unit fractions, whole numbers, and simple cases of fraction equivalence.

In later grades, task **4:4 Comparing Fractions with Equivalence** takes up the second fundamental fraction concept mentioned in "Central math concepts." Task **4:5 Fraction Products and Properties** extends third-grade quantitative thinking about fractions into multiplication, while task **4:7 Fraction Sums and Differences** does something similar for addition, emphasizing the role of unit fractions.

In earlier grades, task **2:14 Correcting a Shape Answer** involves structuring space by partitioning a rectangle and relating partitions to fractional parts (halves).

Additional notes on the design of the task

- Task 3:6 isn't routine; the denominators in the fractions are much greater than those which students commonly study. Drawing fraction models for simpler cases can be useful while investigating the problem, although by design it is unlikely that a single drawing in itself could convincingly settle the question being asked.
- The intent of the task is for students to expand and deepen their understanding through investigation and contemplation. The task is not designed as an opportunity to simply apply a remembered mnemonic or procedure. A general realization about how the size of a unit fraction depends on the size of the numerator can emerge from the investigation and be celebrated as a discovery.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:6? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:6? In what specific ways do they differ from 3:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Fractions make division superfluous from an axiomatic point of view; one may notice that in the properties of operations that govern algebra, there are no properties that refer to division. Division thinking remains important for modeling and problem-solving, even for algebraic problems; the point of principle is that mathematically speaking, any quotient could be rewritten as a product, because of the existence of multiplicative inverses. (The unit fractions in particular are the multiplicative inverses of the nonzero whole numbers.)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?