3:7 Locating Numbers on a Number Line

Teacher Notes

Central math concepts

As noted in the CCSS Standards, a student's conception of number evolves tremendously during the K–8 years:

> At first, "number" means "counting number": 1, 2, 3, ... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.
>
> This ascent through number systems makes it fair to ask: what does the word number mean that it can mean all of these things? One possible answer is that a number is something that can be used to do mathematics: calculate, solve equations, or represent measurements. (p. 58)

The work students do in grade 3 to understand fractions as numbers represents a major development in their mathematical progress. Because fractions have a different format from whole numbers, it may not be apparent that whole numbers and fractions belong to a single family of numbers. But the whole numbers are embedded in the fractions, because $0 = \frac{0}{1}$, $1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $3 = \frac{3}{1}$, and so on. The number line is a useful diagram for representing fractions and whole numbers together as a single family of numbers.

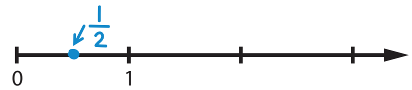
Everyday language works against mathematical language in this area, because in everyday language, "fraction" often carries a connotation of "less than one." (When we say that one purchasing option is "a fraction of the cost" of another option, we don't mean a fraction greater than one.) The everyday connotation of "fraction" as "less than one" could make it counterintuitive that fractions can be greater than one or equal to whole numbers.

The number line is useful for showing equivalences, such as the equivalences $\frac{2}{4} = \frac{1}{2}$ and $\frac{2}{2} = 1$ that appear in task 3:7. Two fractions are equal if they are the same size, or the same point on a number line. In grade 4, this idea leads to the general equivalence principle $\frac{a}{b} = \frac{n \times a}{n \times b}$, which plays a significant role in applying and extending addition, subtraction, multiplication, and division from whole numbers to fractions.

3:7 Here is a list of numbers. Where does each number belong on the number line?

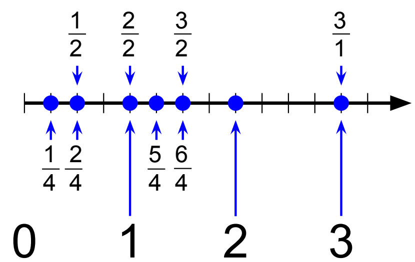
$\frac{1}{2}, \frac{1}{4}, 2, \frac{5}{4}, \frac{2}{2}, \frac{3}{2}, \frac{6}{4}, \frac{2}{4}, \frac{3}{1}$

Draw a dot to show the location of each number. Label each dot. The first number in the list has been located for you.



Answer

See diagram.



Click here for a student-facing version of the task.

Refer to the Standards

3.NF.A; MP.1, MP.2, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The task includes two cases in which a fraction equals a whole number: one in the form $\frac{n}{n} = 1$ and another in the form $\frac{n}{1} = n$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding unit fractions $\frac{1}{2}$ and $\frac{1}{4}$; and using a number line to support and communicate mathematical reasoning.

Extending the task

How might students drive the conversation further?

- Students could discuss and compare the different ways they located each point.
- Students could express 2 as a fraction with a denominator of 2 by continuing the count along the top of the diagram ($\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots$), or express 2 as a fraction with a denominator of 4 by continuing the count below the number line ($\frac{5}{4}, \frac{6}{4}, \ldots$).
- Students could use unit fraction thinking or a story context to make sense of the fact that $\frac{1}{4}$ and $\frac{2}{4}$ are closer together on the number line than $\frac{1}{2}$ and $\frac{2}{2}$.

Related Math Milestones tasks

3:6

3:6 Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

4:4

4:4
(1) Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.
(2) Ariana said, "$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?" Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.
(3) Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

4:5

4:5
(1a–f) Write the values of the products. Compare answers with a classmate.
$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)
(1g) Which answer is twice as much as the answer for (e)?
(1h) Which answer is six times as much as the answer for (a)?
(1i) Which two answers are equal?
(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

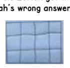
4:7

4:7 Write the values of the expressions. Read each completed equation aloud.
3 fifths + 2 fifths = ___
$\frac{1}{10} + \frac{3}{100} =$ ___ (fraction) $=$ ___ (decimal)
$\frac{6}{25} + \frac{6}{25} =$ ___
$\frac{1}{8} + \frac{5}{8} - \frac{3}{8} =$ ___

5:10

5:10
(1) Solve: $\frac{1}{3} = 0.1 + ?$
(2) Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.
(3) Show one of the above problems and its solution on a number line.

2:14

2:14 Zariah got one answer wrong.
(1) Which answer did Zariah get wrong?
(2) Correct Zariah's wrong answer.
(a) Show how the rectangle can be divided into 15 squares.

(b) 2 halves make one whole.
(c) Draw a triangle. All three sides of your triangle must have different lengths.


2:11

2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?
Draw a diagram to illustrate your solution. Label the diagram with numbers.

Task **3:6 Unit Fraction Ideas** involves the foundational fraction concept, the unit fraction.

In later grades, fraction equivalence and fraction size are the focus in task **4:4 Comparing Fractions with Equivalence**. Operations begin to be extended from whole numbers to fractions in tasks **4:5 Fraction Products and Properties** and **4:7 Fraction Sums and Differences**. Task **5:10 Number System, Number Line** involves fractions and decimals together on the number line.

In earlier grades, task **2:14 Correcting a Shape Answer** involves the concept of one-half as well as spatial structuring that underlies later work with area models. Number lines are useful as diagrams for addition and subtraction problems involving length, for example task **2:11 Grass Snake vs. Rat Snake**.

Additional notes on the design of the task (continued)

- Students don't have to recognize cases of equivalence in advance; the equivalence can emerge from locating a dot where a dot had previously been placed.
- There are many fractions to locate, and this is intended not only to cover several important cases and equivalences but also to create possibilities for using previously located points to inform the choice of where to locate subsequent points. The large number of fractions also begins to suggest the plenitude of numbers themselves (see the Teacher Notes for task **5:10 Number System, Number Line**).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:7? In what specific ways do they differ from 3:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See Zimba (2013), "Units, a Unifying Idea in Measurement, Fractions, and Base Ten."

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?