3:8 Shape Attributes and Categories

Teacher Notes

Math Milestones™

Central math concepts

A distinct thread of geometry learning in the elementary grades deals with shape components, shape properties, and shape categories. The *Progression* document† describes three *levels of geometric thinking* that describe increasing sophistication with this learning progression:

- **Visual/Syncretic level**. Students recognize shapes, for example, a rectangle "looks like a door."
- **Descriptive level**. Students perceive properties of shapes, for example, a rectangle has four sides, all its sides are straight, opposite sides have equal length.
- **Analytic level**. Students characterize shapes by their properties, for example, a rectangle has opposite sides of equal length and four right angles.
- **Abstract level**. Students understand, for example, that a rectangle is a parallelogram because it has all the properties of parallelograms.

By grade 3, as noted in the *Progression* document (p. 13), categories of shapes "can be the raw material for thinking about the relationships between classes. For example, students can form larger, superordinate, categories, such as the class of all shapes with four sides, or quadrilaterals, and recognize that it includes other categories, such as squares, rectangles, rhombuses, parallelograms, and trapezoids. They also recognize that there are quadrilaterals that are not in any of those subcategories." Task 3:8 concentrates on these general notions of shape classifications and relationships between categories of shapes.

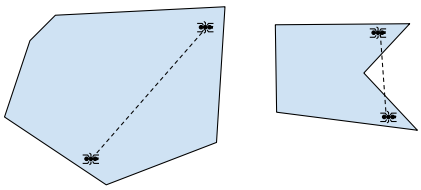
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: remembering definitions of polygons or other two-dimensional shapes; identifying and naming shape attributes; and drawing examples of shapes with defined properties or in defined categories.

Extending the task

How might students drive the conversation further?

- Students could decide, for a given two-dimensional shape, if it is possible for an imaginary tiny ant to travel from any point in the shape to any other point in the shape without ever being outside the shape. In the figure, the shape on the left has this property, but the shape on the right does not.



3:8

(1) Name two attributes that are shared by triangles and squares.

(2) Name a category of shapes that includes triangles and squares and also includes other shapes that have both of the attributes you named.

Answer

(1) Answers may vary, but could include such observations as: triangles and squares both have straight sides; triangles and squares are both closed; triangles and squares are both two-dimensional. **(2)** Possible answers include the category of polygons (which includes hexagons, for example); the category of closed two-dimensional shapes (which includes circles, for example); the category of polygons with fewer than 5 sides (which includes kites, for example).

Click here for a student-facing version of the task.

Refer to the Standards

3.G.A.1; MP.1, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

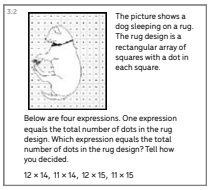
Additional notes on the design of the task

Part (1) of the task directs attention to attributes; part (2) uses the attributes to direct attention to categories.

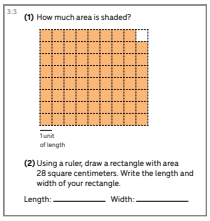
This property defines a category of shapes. What might we call this shape category?‡ Which familiar shapes belong to the category? Could we draw some more shapes that don't belong to the category? Does the category make sense for shapes with curved sides?

Related Math Milestones tasks

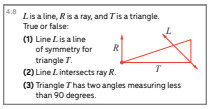
3:2



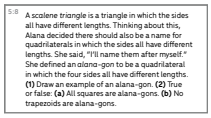
3:3



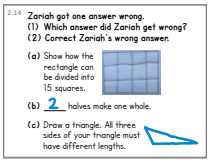
4:8



5:8



2:14



Geometric properties of rectangles are implicit when reasoning about arrays and area; arrays are the context of task **3:2 Hidden Rug Design**, and area is the context of task **3:3 Length and Area Quantities**.

In later grades, task **4:8 Shapes with Given Positions** involves definitions of geometric figures, and task **5:8 Alana's New Shape Category** involves an invented category defined by attributes.

In earlier grades, task **2:14 Correcting a Shape Answer** involves creating a shape with a specified attribute.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 3:8? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 3:8? In what specific ways do they differ from 3:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 3:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 3.

‡ Mathematicians call this category "convex shapes," but students could invent a name for the category, even a whimsical one, based on their understanding of the property.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?