4:10 Calculating Products and Quotients

Teacher Notes

Central math concepts

Task 4:10 focuses on procedures. For the mental calculations in the task, students can use place value, properties of operations, and (in the case of 480 ÷ 8) the relationship between multiplication and division as computation strategies. For the pencil-and-paper calculations in the task, there are various possibilities for efficient, accurate, and generalizable algorithms that handle the given problems.

> **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
>
> **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Computation strategies and computation algorithms are usefully distinguished (CCSS Glossary). Strategies are "purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another." Mental calculation often uses such strategies. For example, we could calculate 5 × 19 mentally by thinking of 5 × 20 - 5, then thinking of 5 × 20 as 5 × 2 × 10, which is 10 × 10 or 100. Thus, the result is 100 - 5 = 95. Alternatively, we could think of 5 × 19 as 50 + 45, obtaining 95 that way. Flexible procedural fluency based on place value, properties of operations, relationships between operations, and ideas of equivalence is valuable both in practical terms and also as a way of doing arithmetic that prefigures algebra. In algebra, strategies are the most common way of working.

Algorithms are inflexible by definition. One step follows another in the prescribed order. Some algorithms are also much simpler to execute than others. The standard multi-digit addition algorithm is less complex than the standard multi-digit subtraction algorithm. The standard algorithm for dividing fractions is less complex than the standard algorithm for dividing multi-digit numbers.

Sometimes even an efficient general-purpose algorithm wouldn't be an efficient approach to a particular instance of a calculation, as in a subtraction problem like 4,003 - 8. On the other hand, when faced with a calculation there may be times when we don't find ourselves readily inventing a flexible mental procedure on the spot, so it's valuable to know and be proficient with an algorithm.

The grade 4 standards (CCSS 4.NBT.B.5, 6) do not set an expectation of fluency for multi-digit multiplication and division; in this grade, multiplying and dividing multi-digit numbers is a substantially conceptual process. (See the Teacher Notes for task **4:2 Multi-Digit Division Concepts**.) However, practice with grade 4 products and quotients can promote confidence while offering opportunities to debug procedures, reinforce ideas, and strengthen recall of single-digit products. (See the Teacher Notes for task **3:12 Products of Single-Digit Numbers** and the Teacher

4:10 Write the values of the products and quotients. Check the quotients by multiplying.

Mentally

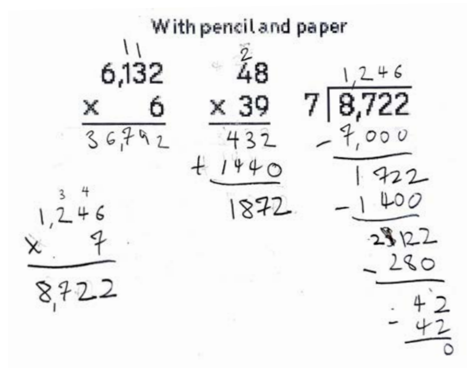
40 × 20
30 × 11
12 × 60
5 × 19
480 ÷ 8

With pencil and paper

6,132 × 6 48 × 39 7)8,722

Answer

For the written calculations, the answers are 36,792; 1,872; and 1,246. For samples of written work, including the multiplication check 1246 × 7 for the problem 8722 ÷ 7, see the examples shown. Students might use different algorithms than the ones shown in the examples.



For the mental calculations, results are shown in **this color**. Some potential thought processes leading to the results (*not the only possible thought processes*) are shown in **this color**. Note that for the mental calculations, it is fine if students manage the mental load by using writing to jot down intermediate results along the way.

40 × 20 = **(4 × 2) × 100 = 8 × 100 = 800**

30 × 11 = **(3 × 11) × 10 = 33 × 10 = 330**

12 × 60 = **10×60 + 2×60 = 600 + 120 = 720**

5 × 19 = **5×20 - 5 = 100 - 5 = 95**

480 ÷ 8 = **(48 ÷ 8) × 10 = 6 × 10 = 60**

Check: 60 × 8 = **(6 × 8) × 10 = 48 × 10 = 480**

Click here for a student-facing version of the task.

[Notes](#) for task **3:13 Fluency within the Multiplication Table**.)

Fluency in particular with multiplying a multi-digit number by a single-digit number, as in the problem 4087 × 3 or the problem 4087 × 5, could help to prepare for fluently multiplying multi-digit numbers by multi digit numbers in grade 5, because of the way a grade 5 calculation like 4087 × 53 involves a sequence of such operations (see figure).

$$\begin{array}{r} 4087 \\ \times \quad 53 \\ \hline 12261 \\ 204350 \\ \hline 216611 \end{array}$$

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value; single-digit products and related quotients; using the commutative, associative, and distributive properties; and the relationship between multiplication and division.

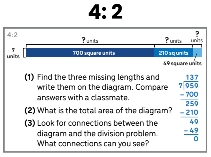
Extending the task

How might students drive the conversation further?

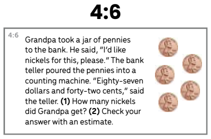
- Students could make sense of their answers by making estimates of the values; for example, 48 × 39 should be somewhat less than 50 × 40 = 2,000.
- Students could help each other debug mistakes that may arise.

Related Math Milestones tasks

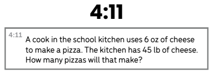
4: 2


4:2
(1) Find the three missing lengths and write them on the diagram. Compare answers with a classmate.
(2) What is the total area of the diagram?
(3) Look for connections between the diagram and the division problem. What connections can you see?

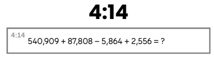
4:6


4:6 Grandpa took a jar of pennies to the bank. He said, "I'd like nickels for this, please." The bank teller poured the pennies into a counting machine. "Eighty-seven dollars and forty-two cents," said the teller. **(1)** How many nickels did Grandpa get? **(2)** Check your answer with an estimate.

4:11

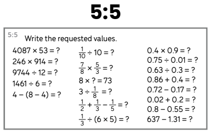

4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

4:14


4:14 540,909 + 87,808 − 5,864 + 2,556 = ?

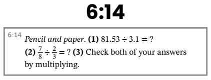
Task **4:2 Multi-Digit Division Concepts** involves multi-digit division from a conceptual point of view. Grade-level multiplication and division calculations could be performed in context for tasks **4:6 Jar of Pennies** and **4:11 School Kitchen**. Task **4:14 Fluency with Multi-Digit Sums and Differences** marks the culmination of the progression for multi-digit addition and subtraction.

5:5


5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10 = ?$	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3} = ?$	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8} = ?$	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5) = ?$	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

6:14


6:14 *Pencil and paper.* **(1)** 81.53 ÷ 3.1 = ? **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

In later grades, task **5:5 Calculating** involves grade-level procedures for addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. Task **6:14 Dividing Decimals and Fractions** marks the culmination of algorithmic procedures with general fraction and decimal division.

Refer to the Standards

4.NBT.B; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

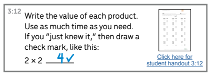
- For the mental calculations, it is fine if students manage the mental load by using writing to jot down intermediate results along the way.
- The task does not require students to show their work, but looking at students' steps can show where they may have made a careless mistake.

Curriculum connection

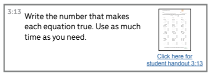
1. In which unit of your curriculum would you expect to find tasks like 4:10? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 4:10? In what specific ways do they differ from 4:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

3:12



3:13



In earlier grades, tasks **3:12 Products of Single-Digit Numbers** and **3:13 Fluency within the Multiplication Table** concern single-digit products and related quotients, which are building blocks for multi-digit multiplication and division algorithms.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?