4:12 Super Hauler Truck

Teacher Notes

Central math concepts

Task 4:12 involves multiplicative comparison, which represents an advance on grade 3 multiplicative thinking. In multiplication and division word problems in grade 3, students multiplied whole numbers to find the total number of objects when the objects were grouped equally or were arrayed in rows and columns (including cases where the objects in rows and columns were square units), and students divided whole numbers to find an unknown factor in such situations (unknown group size, unknown number of groups, or unknown length measure). In grade 4, students will apply and extend their thinking about multiplication and division to solve problems of multiplicative comparison.[†]

Multiplicative comparison problems involve the idea of times-as-many/times-as-much, which is the idea that the product $A \times B$ refers to a quantity that is A times as many/times as much as the quantity B.

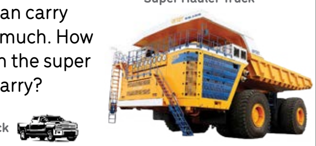
The conceptual or perhaps linguistic difference between times-as-many and times-as-much is that times-as-*many* applies most directly to discrete objects, like bowling balls or boats. Times-as-*much* applies most directly to substances that can be measured out and repeatedly subdivided, like a quantity of fluid or an interval of time. Often, the measures of these substances are fractions. As a conceptual step beyond the idea of equal-groups, the idea of times-as-many/times-as-much may be an important step closer to the grade 5 idea of multiplication as a scaling operation that magnifies or shrinks a quantity.

Extending multiplication and division from whole numbers—and from whole-number-specific mental models like equal groups—to fractions and to the mental models that accommodate fractions, like times-as-much, is perhaps the most important mathematical progression of the upper-elementary grades. The need to support students in making that conceptual evolution raises important questions: What aspects of earlier thinking about multiplying whole numbers will remain helpful when making sense of a product involving fractions? What new ways of thinking will be helpful? And what mathematical representations introduced during whole-number multiplication work are best suited to supporting the transition from multiplying and dividing whole numbers to multiplying and dividing all numbers?

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental math such as $300 \div 5$ or 8×300; and working with mixed numbers and fractions.

4:12 The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?



Answer

480 tons.

Click here for a student-facing version of the task.

Refer to the Standards

4.NF.B.4c, 4.OA.A.2; MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The targeted mathematics in the task is multiplication, not addition. The multiplier of 300 in the task is intentionally large, so as to invite multiplicative thinking.
- Super hauler trucks can, in fact, carry such large payloads; see Haul Truck on Wikipedia.
- In the image for the task, the pickup truck and super hauler truck are shown to actual scale.

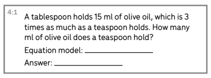
Extending the task

How might students drive the conversation further?

- Students could ask and answer the question, "How many pounds can each truck carry?"
- Students could use the image in the task (which accurately shows the relative sizes of the trucks) to estimate how many times taller or how many times longer the super hauler truck is compared to the pickup truck.

Related Math Milestones tasks

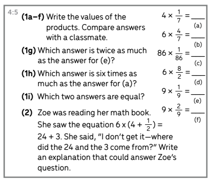
4:1


4:1 A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

Equation model: ____________

Answer: ____________

4:5


4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ____ (a)

$6 \times \frac{4}{7} =$ ____ (b)

$86 \times \frac{1}{86} =$ ____ (c)

$6 \times \frac{8}{2} =$ ____ (d)

$9 \times \frac{1}{9} =$ ____ (e)

$9 \times \frac{2}{9} =$ ____ (f)

(1g) Which answer is twice as much as the answer for (e)?

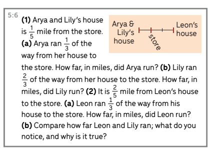
(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

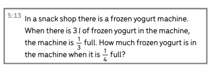
Task **4:1 A Tablespoon of Oil** is a word problem involving multiplicative comparison (situation type Compare with Smaller Unknown). Task **4:5 Fraction Products and Properties** concentrates on the concepts of multiplying a fraction by a whole number.

5:6

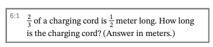


5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

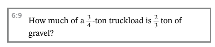
5:13


5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

6:1

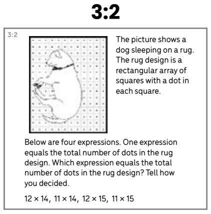

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9


6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

In later grades, task **5:6 Corner Store** involves products of unit fractions and products of other fractions in context, and task **5:13 Frozen Yogurt Machine** involves both multiplying and dividing with fractions. Tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** are word problems that involve finding an unknown factor in a fraction product.

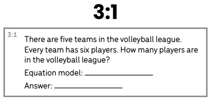
3:2


3:2 The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12×14, 11×14, 12×15, 11×15

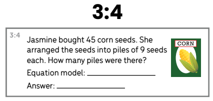
3:1


3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: ____________

Answer: ____________

3:4

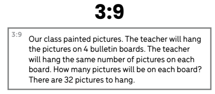


3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

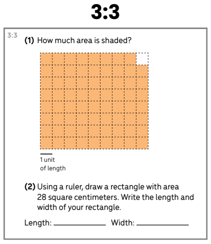
Equation model: ____________

Answer: ____________

3:9


3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:3



3:3 **(1)** How much area is shaded?

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ____________ Width: ____________

In earlier grades, task **3:2 Hidden Rug Design** centers on the equal-groups concept of multiplication in an array context, while tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, and **3:9 Bulletin Board Pictures** are word problems centered on equal-groups concepts of multiplication and/or division. Multiplication is useful in task **3:3 Length and Area Quantities**.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:12? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:12? In what specific ways do they differ from 4:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See Table 3, p. 23 of *Progressions for the Common Core State Standards in Mathematics (draft). Grade 8, High School, K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?