4:13 Area Units

Teacher Notes

Central math concepts

To work successfully with measured quantities, and also to reason with place value and fractions, students must attend to units.† For example, suppose we have a roll of 10 **yards** of packing tape and another roll of 30 **feet** of packing tape. Then the total length of packing tape isn't 10 + 30 = 40 yards. Nor is the total length 10 + 30 = 40 feet.‡

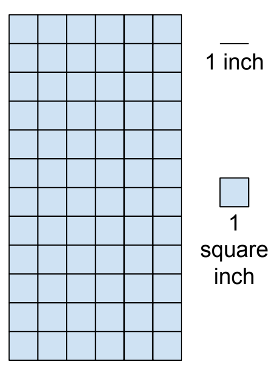
If we measure the same quantity using different units, then the numerical value of the measure will be different, even though the quantity itself is the same. For example, the length of a snow leopard's tail might have the value 36 when measured in units of inches, and a value of 3 when measured in units of feet. But it's the same tail. An inverse multiplicative relationship is present here: a foot is 12 times longer than an inch, so it takes 12 times as many inches as feet to measure the same length.

In grades K–2, students work with length units and concepts of length measurement (CCSS 1.MD.A.2). In kindergarten, students make non-numerical comparisons of length and other measurable quantities (CCSS K.MD.A). In grade 1, using objects as length units, students learn that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. In grade 2, students extend these ideas to abstract length units and relate length measurement to addition, subtraction, and the number line (CCSS 2.MD.A).

After studying length as a measurable quantity in the primary grades, students learn to recognize area as a measurable attribute of plane figures. Area is the amount of two-dimensional surface a figure contains. Consistent with this idea, congruent figures are assumed to enclose equal areas.

Students also understand the concepts involved in measuring area (CCSS 3.MD.C.5):

- **A unit of measure for area:** An area unit is built from a chosen length unit. Given a length unit, a square with side length equal to 1 unit, called "a unit square," is said to have "one square unit" of area.
- **Quantifying area:** A plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.



For example, if a rectangle has length 12 inches and width 6 inches, then 72 square inches will cover the rectangle with no gaps or overlaps (see figure).

In task 4:13, if the length unit is 1 inch, then applying the formula $A = L \times W$ to the red rectangle leads to the calculation $A = 6 \times 12 = 72$. If the length unit is 1 foot, then applying the formula $A = L \times W$ to the blue rectangle leads to the calculation $A = 1 \times \frac{1}{2} = \frac{1}{2}$. The numbers 72 and $\frac{1}{2}$ are unequal,

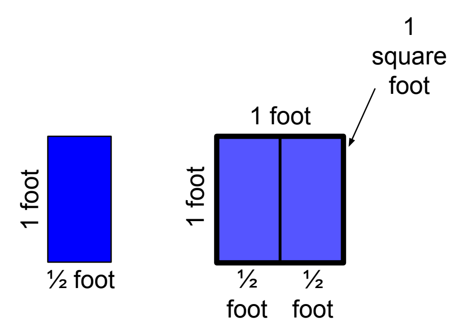
4:13 **(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

Answer

(1) 72 in^2. **(2)** See example picture; the picture shows that the area of the blue rectangle is half of a square foot, or $\frac{1}{2}$ ft^2. **(3)** Yes. Explanations may vary but could include the idea that the red and blue rectangles can be laid atop one another exactly; that 72 square inches can tile half of a square foot; or that 144 square inches can tile one square foot.



Click here for a student-facing version of the task.

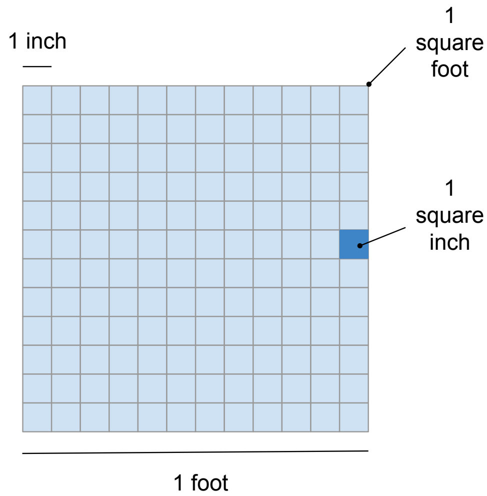
Refer to the Standards

4.MD.A.3; MP.3, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

but 72 in^2 and $\frac{1}{2}$ ft^2 refer to equal quantities of area. The number 72 is 144 times as much as $\frac{1}{2}$, and this is a consequence of the fact a square foot has 144 times as much area as a square inch (see figure).



Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: spatially structuring a rectangle into an array; multiplying a two-digit number by a single-digit number; using a formula; and working with measurement concepts.

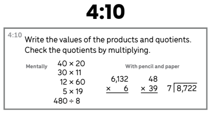
Extending the task

How might students drive the conversation further?

- Students could explore what the area measure of the red rectangle would be if a new, smaller unit of area were chosen, such as a square with side length $\frac{1}{4}$ inch. (Compared to their length measures in inches, the length measures of the rectangle when using the smaller unit are both increased by a factor of 4, so the area formula implies a total increase in the area by a factor of 16. From another perspective, each square inch contains 16 of the new smaller units, so the number of the smaller units is 16 times greater.)
- Students could discuss the situation described in "Central math concepts" in which we have one roll of 10 yards of packing tape and another roll of 30 feet of packing tape. How would we assign a numerical measure to the total length of packing tape?

Related Math Milestones tasks

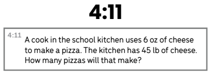
4:10


4:10 Write the values of the products and quotients. Check the quotients by multiplying.

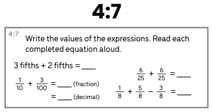
Mentally: 40×20, 30×11, 12×60, 5×19, $480 \div 8$

With pencil and paper: $6{,}132 \times 6$, 48×39, $8{,}722 \div 7$

4:11


4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

4:7


4:7 Write the values of the expressions. Read each completed equation aloud.

3 fifths + 2 fifths = ___

$\frac{1}{10} + \frac{3}{100}$ = ___ (fraction) = ___ (decimal)

$\frac{6}{25} + \frac{6}{25}$ = ___

$\frac{1}{8} + \frac{5}{8} - \frac{3}{8}$ = ___

Like part (1) of task 4:13, tasks **4:10 Calculating Products and Quotients** and **4:11 School Kitchen** both involve products outside of the 10 × 10 multiplication table. Like part (2) of task 4:13, task **4:7 Fraction Sums and Differences** involves addition of fractional quantities.

Additional notes on the design of the task

The task does not include a diagram, and that is intentional because thoughtfully building the diagrams is part of the task.

Curriculum connection

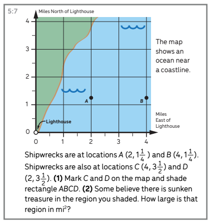
1. In which unit of your curriculum would you expect to find tasks like 4:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:13? In what specific ways do they differ from 4:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Even adding apples and oranges requires introducing a new unit, such as "fruit." See Teacher Notes for task **K:14 Animals from Land and Sea**. Brief observations and examples of the role of units throughout arithmetic can be found in "Units, a Unifying Idea in Measurement, Fractions, and Base Ten" (blog post by Jason Zimba, 2013).

‡ Is the total amount of packing tape equal to 40 of any length unit? What length unit?

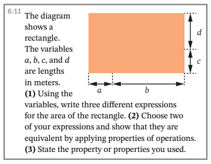
* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

5:7



Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

6:11



The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters. **(1)** Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

6:12

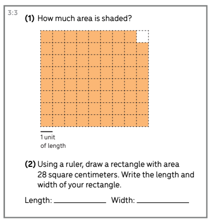
6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

7:7

7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

In later grades, task **5:7 Shipwrecks** involves rectangle area in context, for a rectangle with fractional dimensions, and task **6:11 Area Expressions** involves rectangle area in a case where the lengths are variables rather than numbers. Task **6:12 Coordinate Triangle** involves area measure for a triangle. Task **7:7 Speed Limit** involves comparing two quantities of speed given in different units.

3:3



(1) How much area is shaded?

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ____________ Width: ____________

In earlier grades, task **3:3 Length and Area Quantities** involves concepts of area measurement.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?