4:14 Fluency with Multi-Digit Sums and Differences

Teacher Notes

Central math concepts

Task 4:14 focuses on fluency with procedures and, in particular, fluency with the standard algorithms for multi-digit addition and subtraction. In these algorithms, one sets up a vertical tableau and works from right to left, with the result appearing immediately below the horizontal line.

$$\begin{array}{rr} & 540909 \\ & 87808 \\ + & 2556 \\ \hline & 631273 \end{array} \qquad \begin{array}{rr} & 631273 \\ - & 5864 \\ \hline & 625409 \end{array}$$

> **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
>
> **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Algorithms are usefully distinguished from strategies (CCSS Glossary; see figure). Strategies are "purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another." Mental calculation often uses strategies. For example, we could calculate 205 - 9 mentally by replacing the given difference with the equivalent problem 205 - 5 - 4. Flexible procedural fluency based on place value, properties of operations, relationships between operations, and ideas of equivalence is valuable both in practical terms and also as a way of doing arithmetic that prefigures algebra. In algebra, strategies are the most common way of working.

Algorithms are inflexible by definition. One step follows another in the prescribed order. However, there are still some choices in task 4:14, such as whether to organize the work as shown in the figure above, or in another way (such as by adding the first two numbers, then subtracting the second from the sum, then adding the last number to the difference; or such as by adding the first two numbers, then subtracting from that result the difference between 5,864 and 2,556).

The efficiency of the standard multi-digit algorithms comes from the fact that the place value system is uniform from one place to another. Yet sometimes even an efficient general-purpose algorithm wouldn't be an efficient approach to a particular instance of a calculation, as in a subtraction problem like 4,003 - 8. On the other hand, when faced with a calculation, there may be times when we don't find ourselves readily inventing a flexible mental procedure on the spot, so it's valuable to know and be proficient with an algorithm.

An important value in mathematics education is that of being able to solve problems in multiple ways. This brings the pleasures of seeing how a coherent subject holds together, and it allows students to check answers, unify their understanding of concepts, and learn from different ways of thinking that emerge in the classroom community. A parallel but also important outcome of mathematics education is for students to be

4:14 540,909 + 87,808 – 5,864 + 2,556 = ?

Answer

625,409.

Click here for a student-facing version of the task.

Refer to the Standards

4.NBT.B.4; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

The task does not require students to show their work, but looking at students' steps can show where they may have made a careless mistake.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:14? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 4:14? In what specific ways do they differ from 4:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

supported in gaining procedural fluency with algorithms for the actually quite small set of recurrent problem types for which an algorithm exists. This small set can be found in the CCSS-M by searching for "algorithm."

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value concepts; and single-digit sums and differences.

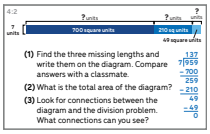
Extending the task

How might students drive the conversation further?

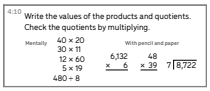
- Checking differences by adding can offer additional procedural practice and reinforce the relationship between addition and subtraction ($C - A$ is the unknown factor in $A + \square = C$).
- Students could make sense of their answers another way by making estimates of the values; for example, the answer in thousands should be reasonably close to 541 + 88 - 3 = 541 + 85 = 626.

Related Math Milestones tasks

4:2



4:10



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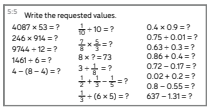
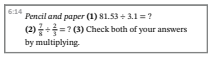
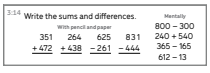


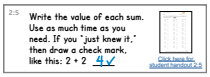
6:14



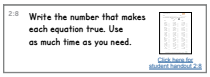
3:14



2:5



2:8



Task **4:2 Multi-Digit Division Concepts** involves another multi-digit algorithm, but at a pre-fluency stage in the learning progression. Task **4:10 Calculating Products and Quotients** involves grade-level procedures with multi-digit multiplication and division.

In later grades, task **5:5 Calculating** continues procedures into larger numbers of digits and into fractions and decimals. Task **6:14 Dividing Decimals and Fractions** marks the culmination of algorithmic procedures with general fraction and decimal division.

In earlier grades, task **3:14 Fluency within 1000 (Add/Subtract)** involves grade-level fluencies for addition and subtraction, while tasks **2:5 Sums of Single-Digit Numbers** and **2:8 Fluency within the Addition Table** involve the single-digit sums and related differences upon which multi-digit addition and subtraction algorithms are built.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?