4:1 A Tablespoon of Oil

Teacher Notes

Math Milestones™

Central math concepts

Based on interviews with middle-grades students,† education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract…; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "…if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

The only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations, so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

For multiplication and division specifically, the mathematical relationship between the operations is that $C \div A$ is the unknown factor in $A \times ? = C$. Therefore, problems involving division also implicitly involve multiplication, because division finds an unknown factor. This is why a division calculation is checked by multiplying.

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between multiplication and division can play out in situations in conceptually distinct ways, including:‡

4:1 A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

Equation model: ______________

Answer: ______________

Answer

$3 \times m = 15$, $15 \div 3 = m$, or another equivalent equation. A teaspoon holds 5 ml of olive oil.

Click here for a student-facing version of the task.

Refer to the Standards

4.OA.A.2; MP.1, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

The numbers 15 and 3 in the task are intended to make the fluency demands of the task low so that attention can focus on the relationships in the situation.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:1? In what specific ways do they differ from 4:1?

- **Equal Groups:** Product Unknown, Group Size Unknown, and Number of Groups Unknown
- **Arrays:** Product Unknown, Number of Rows Unknown, and Number of Columns Unknown
- **Compare:** Size of Larger Quantity Unknown, Size of Smaller Quantity Unknown, and Multiplier Unknown

In particular, the situation type in task 4:1 is called "Compare with Smaller Unknown." It is a Compare situation because multiplication is being used to compare the capacity of a tablespoon with the capacity of a teaspoon. And the situation is "Compare with Smaller Unknown" because the initially unknown quantity is the capacity of a teaspoon (which is less than the capacity of a tablespoon).

Multiplicative comparison represents an advance on grade 3 multiplicative thinking. In multiplication and division word problems in grade 3, students multiplied whole numbers to find the total number of objects when the objects were grouped equally or were arrayed in rows and columns (including cases where the objects in rows and columns were square units), and students divided whole numbers to find an unknown factor in such situations (unknown group size, unknown number of groups, or unknown length measure). In grade 4, students will apply and extend their thinking about multiplication and division to solve problems of multiplicative comparison. These problems involve the idea of times-as-many/times-as-much, which is the idea that the product $A \times B$ refers to a quantity that is A times as many/times as much as the quantity B.

As a conceptual step beyond the idea of equal-groups, the idea of times-as-many/times-as-much may be an important step closer to the grade 5 idea of multiplication as a scaling operation that magnifies or shrinks a quantity. The conceptual or perhaps linguistic difference between times-as-many and times-as-much is that times-as-*many* applies most directly to discrete objects, like bowling balls or boats. Times-as-*much* applies most directly to substances that can be measured out and repeatedly subdivided, like a quantity of fluid or an interval of time. Often, the measures of these substances are fractions. Depending on the context, times-as-much language could involve phrases like "as long as," "as far as," "as tall (or short) as," "as heavy as," "as expensive as," and so on.

Task 4:1 asks not only for the final answer but also for an equation model. An equation model is requested because compared to the answer alone ("5 ml"), an equation model is better evidence that students have comprehended the situation and its quantitative relationships. The equation model records the situation's mathematical structure so that students can discuss and reflect on it. Equivalent forms of the equation can also show the relationship between multiplication and division.

It may happen that not all students write the same equation. That is an opportunity to discuss correspondences between equations and relationships to the situation described in the problem. The *Progression* document stresses that "[i]f textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ See Table 3, p. 23 of *Progressions for the Common Core State Standards in Mathematics (draft)*: *K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

representation to a context, and not the representation separated from its context" (p. 13).

Some equation models describe a situation in an algebraic way, such as $3 \times m = 15$, (these are called *situation equations*); students may then rewrite a situation equation in a form more conducive to determining the unknown number, such as $15 \div 3 = m$ (this is called a *solution equation*). As observed in the *Progression* document,

> Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra. (p. 13)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: recall of single-digit products; writing situation equations and solution equations; and using a tape diagram or other representation.

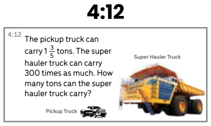
Extending the task

How might students drive the conversation further?

- Students who wrote different equation models could compare them and identify correspondences between the equations and each other, and the equations and the quantities and relationship in the situation.
- Students could discuss the claim that a teaspoon holds $\frac{1}{3}$ of what a tablespoon holds, basing their discussion on number lines or tape diagrams.

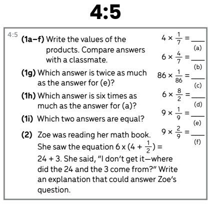
Related Math Milestones tasks

4:12



4:12 The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?

4:5



4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)

$6 \times \frac{4}{7} =$ ___ (b)

$86 \times \frac{1}{86} =$ ___ (c)

$6 \times \frac{8}{2} =$ ___ (d)

$9 \times \frac{1}{9} =$ ___ (e)

$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

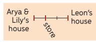
(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

Task **4:12 Super Hauler Truck** is a word problem involving multiplicative comparison (the situation type is Compare with Larger Unknown; see Table 3, p. 23 of the *Progression* document). Task **4:5 Fraction Products and Properties** concentrates on the concepts of multiplying a fraction by a whole number, a step along the way to multiplying fractions in grade 5 and the idea of multiplication as scaling.

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



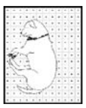
5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

In later grades, task **5:6 Corner Store** involves products of unit fractions and products of other fractions in context, and **task 5:13 Frozen Yogurt Machine** involves both multiplying and dividing with fractions.

3:2

3:2



The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

$12 \times 14,\ 11 \times 14,\ 12 \times 15,\ 11 \times 15$

3:1

3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: ____________

Answer: ____________

3:4

3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: ____________

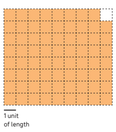
Answer: ____________

3:9

3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

3:3

3:3 **(1)** How much area is shaded?



(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ____________ Width: ____________

In earlier grades, task **3:2 Hidden Rug Design** centers on the equal-groups concept of multiplication in an array context, while tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, and **3:9 Bulletin Board Pictures** are word problems centered on equal-groups concepts of multiplication and/or division. Multiplication is useful in task **3:3 Length and Area Quantities**.

4:1 A Tablespoon of Oil

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?