4:2 Multi-Digit Division Concepts

Teacher Notes

Central math concepts

Asked to calculate the quotient 959 ÷ 7, a mischievous person might immediately answer, “Nine hundred fifty-nine sevenths.” That answer is correct! Unfortunately, it doesn’t tell us everything about the quotient we might need to know, such as what its tens digit is. Long division and its variations are useful algorithms for expressing the quotient of two multi-digit numbers as a multi-digit number. These algorithms also present opportunities for practicing and deepening student understanding of earlier mathematics such as place value to hundreds.

In part (1) of task 4:2, the dividend 959 is represented as the measure of a rectangular area; the divisor 7 is represented as a length measure for one dimension of the rectangle. Then the quotient, 137, corresponds to the length measure of the other dimension of the rectangle. Area ÷ *length* = *length*. If we view the rectangle with an unknown total length as a missing factor problem, 7 × ? = 959, then the unknown factor is the quotient 959 ÷ 7. Finding the base-ten digits of that quotient is what the division problem in part (3) is doing.

The distributive property $a \times (b + c) = a \times b + a \times c$ is fundamental not only to multiplying multi-digit numbers, but also to dividing them. In task 4:2, the division algorithm has parceled 959 into a sum of three terms, 959 = 700 + 210 + 49. The three terms have a common factor of 7:

$$959 = 7 \times 100 + 7 \times 30 + 7 \times 7.$$

Applying the distributive property to the right-hand side, we obtain

$$959 = 7 \times (100 + 30 + 7)$$

from which we can read the digits of the quotient, 137. We could express the multiplicative relationships above using division, too:

$$959 \div 7 = (700 + 210 + 49) \div 7$$

$$959 \div 7 = 700 \div 7 + 210 \div 7 + 49 \div 7$$

and this is a sense in which division could be said to distribute over addition.

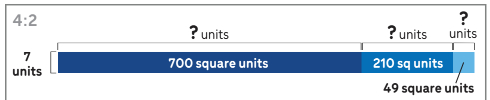
An interesting feature of the standard long division algorithm is that it returns the base-ten digits in succession at each successive step. However, from the standpoint of the distributive property, nothing requires that the decomposition of 959 be in base-ten units. The distributive property is true independently of the base-ten system for writing numbers. So for example, we might have

$$959 \div 7 = (350 + 420 + 189) \div 7$$

$$\begin{aligned} 959 \div 7 &= 350 \div 7 + 420 \div 7 + 189 \div 7 \\ &= 50 + 60 + 27 \\ &= 137. \end{aligned}$$

The partial quotients algorithm makes use of this flexibility in the distributive property.

4:2



(1) Find the three missing lengths and write them on the diagram. Compare answers with a classmate.

(2) What is the total area of the diagram?

(3) Look for connections between the diagram and the division problem. What connections can you see?

$$\begin{array}{r} 137 \\ 7\overline{)959} \\ -700 \\ \hline 259 \\ -210 \\ \hline 49 \\ -49 \\ \hline 0 \end{array}$$

Answer

(1) From left to right, the missing lengths are 100 units, 30 units, and 7 units. **(2)** 959 square units. **(3)** Answers will vary. Possible connections between the diagram and the division problem could include that the height of the rectangle is 7 units, and the divisor is 7; that the area measures in square units (700, 210, and 49) appear in the algorithm steps as partial products; that those partial products are the same products that give the shaded areas (7 × 100, 7 × 30, and 7 × 7); that the quotient is the sum of the length measures 100, 30, and 7; that the total area of the rectangle is 700 + 210 + 49 = 959, which is the dividend; that the algorithm step 959 - 700 = 259 corresponds to subtracting the darkest-shaded area from the total area.

Click here for a student-facing version of the task.

Refer to the Standards

4.NBT.B.6; MP.2, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Eventually, the kinds of reasoning processes involved in task 4:2 will cease to be the focus of a student's work with multi-digit division, and calculating quotients of whole numbers will become a routine exercise in procedural fluency. That progression takes time, however, and it requires a healthy mix of fluency practice coordinated with reasoning and conversational sense-making.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value to hundreds; multiplying with multiples of ten; multiplying by a single-digit factor; subtracting with three-digit numbers; and reasoning based on area concepts and the distributive property.

Extending the task

How might students drive the conversation further?

- If students normally calculate a quotient like 959 ÷ 7 using a different algorithm than the one shown (for example, partial quotients), they could ask if there is a way to change the diagram so that it connects to their work. For example, the boundaries between the shaded regions could be moved, and/or additional boundaries could be created.
- Students could ask how the diagram might be changed to match a problem with a remainder; the problem 960 = 7 × 137 + 1 could be considered.
- Students could brainstorm the most common careless errors ("bugs") a student might make in long division. Then students could formulate a tip for avoiding or finding and correcting the error in other long division problems.

Related Math Milestones tasks

4:11

4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

4:6

4:6 Grandpa took a jar of pennies to the bank. He said, "I'd like nickels for this, please." The bank teller poured the pennies into a counting machine. "Eighty-seven dollars and forty-two cents," said the teller. **(1)** How many nickels did Grandpa get? **(2)** Check your answer with an estimate.

4:10

4:10 Write the values of the products and quotients. Check the quotients by multiplying.

Mentally: 40 × 20; 30 × 11; 12 × 60; 5 × 19; 480 ÷ 8

With pencil and paper: 6,132 × 6; 48 × 39; 7)8,722

Task **4:11 School Kitchen** could involve finding a quotient of a three-digit dividend (which is a multiple of ten) with a one-digit divisor. Task **4:6 Jar of Pennies** could involve finding a quotient or whole-number quotient of a four-digit dividend and a one-digit divisor. Task **4:10 Calculating Products and Quotients** includes a multi-digit division problem.

5:5

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10 = ?$	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3} = ?$	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8} = ?$	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5) = ?$	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

5:10

5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$

(2) Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.

(3) Show one of the above problems and its solution on a number line.

In later grades, task **5:5 Calculating** includes multi-digit division problems. Task **5:10 Number System, Number Line** (part (1)) involves a problem in which one operates with numbers that are initially given in different formats (one fraction, one decimal).

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

To prompt students to make connections, students might first be asked to circle a three-digit number that they can see in both the diagram and the algorithm.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:2? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:2? In what specific ways do they differ from 4:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

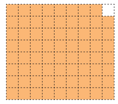
3:13

3:13 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 3:13

3:3

3:3 **(1)** How much area is shaded?



1 unit of length

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ___________ Width: ___________

In earlier grades, task **3:13 Fluency within the Multiplication Table** involves mental calculation of quotients related to single-digit products. Task **3:3 Length and Area Quantities** involves concepts of area.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?