4:3 Pencil Data

Teacher Notes

Math Milestones™

Central math concepts

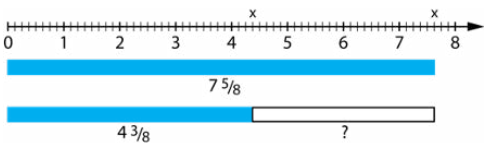
Students' data work in the upper-elementary grades concentrates on measurement data displayed in line plots.[†] In a line plot like the one shown in task 4:3, the "x" symbols are the individual data points.[‡]

The partial number line diagram in a line plot corresponds to the scale on the measurement tool that was used to generate the data. In task 4:3, the measurements are lengths in inches, but if the measurements had been liquid volumes, for example, then the units on the scale would be liters or another unit of liquid volume.

As for the vertical scale on a line plot, a vertical scale isn't shown on the line plot in task 4:3, but if a vertical scale had been shown, then it would be a count scale, meaning that the tick marks on the vertical scale would be the numbers 0, 1, 2, 3, and so on, indicating the number of observations above each tick mark.

Interpreting a line plot involves grasping the correspondence between an "x" symbol or dot, its horizontal position on the measurement scale, and what fact about the situation is being thereby recorded. For example, the leftmost "x" symbol records the fact that one of the pencils was $4\frac{3}{8}$ inches long.

From the individual data points (the "x" symbols), students can use counting to determine the total number of observations, 25, which is one numerical summary of the data (part (1)).



Students' work in representing and analyzing measurement data connects directly to their growing number sense of fractions and to their increasing ability to use addition and subtraction with fractions to solve problems in context. In task 4:3, students use a partial number line diagram marked in eighths. They use subtraction to determine the answer to a Compare word problem (part (2); see figure). And they use addition and/or subtraction to determine the answer to a Put Together/Take Apart word problem with Both Addends Unknown. (See Table 2 in the relevant Progression document.[§]) There are close connections in every elementary grade between students' data work and their expanding use of numbers and operations in context; see Table 1, p. 4 of the relevant *Progression* document for a list of these connections in grades K–5.

As the *Guidelines for Assessment and Instruction in Statistics Education Report* notes, "data are not just numbers, they are numbers with a context. In mathematics, context obscures structure. In data analysis, context provides meaning."[¶] Thus as the *Progression* document notes, "students should work with data in the context of science, social science, health, and other subjects, always interpreting data plots in terms of the context they represent" (p. 3).

4:3

Everyone in class measured the length of their pencil. Here are the measurements:

(1) How many pencils were measured?

(2) How much longer was the longest pencil than the shortest pencil?

(3) Could two of the pencils be laid end to end to make a total length of 1 foot?

Answer

(1) 25. **(2)** $3\frac{1}{4}$ inches (or $3\frac{2}{8}$ inches). (The longest pencil was $7\frac{5}{8}$ inches long, and the shortest pencil was $4\frac{3}{8}$ inches long; $7\frac{5}{8} - 4\frac{3}{8} = 3\frac{2}{8}$.) **(3)** Yes, because $4\frac{3}{8} + 7\frac{5}{8} = 11 + \frac{8}{8} = 12$.

Click here for a student-facing version of the task.

Refer to the Standards

4.MD.B.4; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The situation types involved in the task are "Compare with Difference Unknown" and "Put Together/Take Apart with Both Addends Unknown." Students first encounter these situations with whole numbers in the primary grades, and they revisit them with fractional quantities in the upper-elementary grades.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using number lines with fractions; calculating sums and differences of mixed numbers with equal denominators; and using addition and subtraction to solve problems in context.

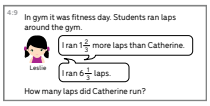
Extending the task

How might students drive the conversation further?

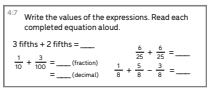
- Students could ask additional questions about the data, such as "How long would all the pencils be if they were all laid end to end?"
- Students could consider what the resulting line plot display of the data might look like in a week or two, imagining that everyone keeps their pencil but sharpens it from time to time.

Related Math Milestones tasks

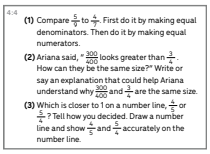
4:9



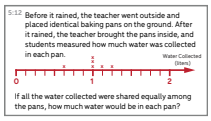
4:7



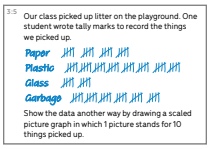
4:4



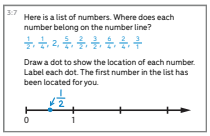
5:12



3:5



3:7



Task **4:9 Fitness Day** involves a Compare situation with mixed-number quantities, as task 4:3 does. Task **4:7 Fraction Sums and Differences** involves fraction calculations with equal denominators, with a connection to unit thinking. Fractions on a number line are involved in task **4:4 Comparing Fractions with Equivalence** (part (3)).

In later grades, task **5:12 Rain Measurements** involves a line plot for measurement data and a calculation that prefigures measures of center and the study of distributions in grade 6.

In earlier grades, task **3:5 Playground Cleanup** focuses on representing a set of categorical data, which is the other major data type in the elementary grades. Task **3:7 Locating Numbers on a Number Line** involves whole numbers and fractions, including fractions equal to whole numbers.

Additional notes on the design of the task (continued)

- Part (1) is intended to orient students to the situation and to the connection between the line plot and the facts which it records.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:3? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:3? In what specific ways do they differ from 4:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, June 20). *Progressions for the Common Core State Standards in Mathematics (draft): K–3, Categorical Data; Grades 2–5, Measurement Data* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ In line plots generated by technology, data points are often marked by small filled circles, or "dots." Apart from that cosmetic feature, the terms *line plot* and *dot plot* are synonymous.

§ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

¶ The Guidelines for Assessment and Instruction in Statistics Education Report was published in 2007 by the American Statistical Association, http://www.amstat.org/education/gaise.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?