4:4 Comparing Fractions with Equivalence

Teacher Notes

Math Milestones™

Central math concepts

The fraction equivalence principle $\frac{a}{b} = \frac{n \times a}{n \times b}$ is one of the most powerful reasoning tools and procedural tools in students' study of fractions. Far from being about "cancelling" at this grade, the fact that $\frac{a}{b}$ and $\frac{n \times a}{n \times b}$ have the same value (are the same size, occupy the same point on a number line) is a reasoned conclusion at this grade, based on conceptual understanding of the inverse relationship between the sizes of the parts $\frac{1}{b}$ and $\frac{1}{n \times b}$ and the number of those parts (a and $n \times a$)†. Part (2) of task 4:4 centers most directly on this multiplicative relationship, since it includes two equivalent fractions with a readily apparent multiplying factor in the numerator and denominator, $\frac{3}{4}$ and $\frac{300}{400}$. Meanwhile part (1) of the task leverages the power of the equivalence principle to compare two fractions in a case where using benchmark fractions would be difficult. Part (3) of the task uses the number line as the setting for integrating two essential fraction concepts: fraction size and fraction location on the number line. Students can also use fraction equivalence to help settle the question in this part of the task.

Students are often taught to remember procedures for comparing fractions, such as by "cross-multiplying" and comparing the resulting whole-number products. From the standpoint of mathematics as a discipline, procedures like that can be antithetical to the disciplinary values of mathematics if the rules aren't authentically accessible to justification and deliberation by students. From the standpoint of equity, rules and recipes delivered from "on high" devolve mathematical authority away from students and can thereby damage an equitable discourse community. From the standpoint of pragmatism, on-demand procedural fluency with exotic fraction comparisons just isn't important for elementary-school students. From the standpoint of achievement, rules that aren't understood can be harder to remember when needed. Finally, from the standpoint of mathematical content, reasoning about fractions ought to cast fractions in the starring role, whereas cross-multiplying concentrates attention on whole numbers. Observe that the answers to part (1) of task 4:4 are statements about fractions.

- The comparison $\frac{35}{63} < \frac{36}{63}$ involves the idea that 35 of the quantity $\frac{1}{63}$ is less than 36 of that same quantity.
- The comparison $\frac{20}{36} < \frac{20}{35}$ involves the idea that $\frac{1}{36}$ is less than $\frac{1}{35}$, and 20 of a smaller quantity is less than 20 of a larger quantity.

Whole numbers here are serving as multipliers of unit fractions, so that for example the phrase "35 of a quantity $\frac{1}{63}$" refers to a fractional quantity. This is different from saying $\frac{5}{9} < \frac{4}{7}$ "because $5 \times 7 < 9 \times 4$"...a statement

4:4

(1) Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.

(2) Ariana said, "$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?" Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.

(3) Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

Answer

(1) *Equal denominators comparison*: $\frac{5}{9} < \frac{4}{7}$ because $\frac{35}{63} < \frac{36}{63}$. (This comparison involves the idea that 35 of the quantity $\frac{1}{63}$ is less than 36 of that same quantity.) *Equal numerators comparison*: $\frac{5}{9} < \frac{4}{7}$ because $\frac{20}{36} < \frac{20}{35}$. (This comparison involves the idea that $\frac{1}{36}$ is less than $\frac{1}{35}$, and 20 of a smaller quantity is less than 20 of a larger quantity.)

(2) Answers will vary. One kind of explanation involves the idea that 3 of a quantity that is a hundred times greater equals 300 of a quantity that is a hundred times smaller. Answers may include such explanatory techniques as emphasizing the meaning of the numerator and denominator in $\frac{300}{400}$; creating a simple context for the numbers in Ariana's comparison (for example, $\frac{1}{4}$ of \$400 is \$100, so $\frac{3}{4}$ is \$300, and $\frac{1}{400}$ of \$400 is \$1, so $\frac{300}{400}$ is \$300); and writing an explanation based on a drawn number line that indicates (conceptually) the equivalence between $\frac{3}{4}$ and $\frac{300}{400}$.

about whole numbers that is unattached to the fraction quantities in the problem or their unit fractions.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using unit fractions; recall of single-digit products; and basing reasoning on math diagrams.

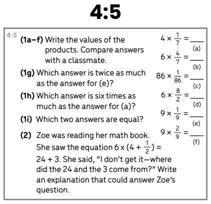
Extending the task

How might students drive the conversation further?

- Students could notice in part (1) that the numbers 35 and 36 "made all the difference" in comparing $\frac{5}{9}$ and $\frac{4}{7}$, using both the numerator approach and the denominator approach. Can this observation be generalized? When comparing $\frac{19}{25}$ and $\frac{16}{21}$ using the numerator approach and the denominator approach, what numbers "make all the difference"?
- Students could consider another version of Ariana's question in part (2), this time involving the fractions $\frac{377}{477}$ and $\frac{3}{4}$. Are these fractions equivalent? Why or why not?
- Building on the conclusions of part (3), students could consider additional cases that fit the pattern of $\frac{4}{5}$ and $\frac{5}{4}$, such $\frac{2}{3}$ and $\frac{3}{2}$, $\frac{7}{8}$ and $\frac{8}{7}$, etc. Can the conclusion of part (3) be generalized?

Related Math Milestones tasks

4:5



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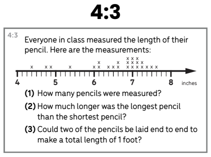

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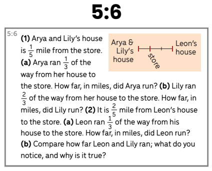

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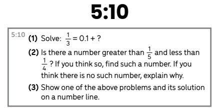

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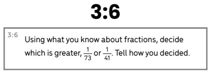


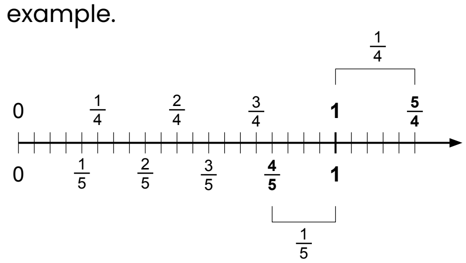
Fraction equivalence sheds light on some parts of task **4:5 Fraction Products and Properties** (such as the fact that $6 \times \frac{8}{2} = 6 \times 4$). A number line appears in task **4:3 Pencil Data** as a measurement scale, and number lines could be used to discuss the mathematics in several other tasks.

In later grades, fraction equivalence could be used to solve or illuminate the mathematics in several tasks, for example, **5:6 Corner Store**. Task **5:10 Number System, Number Line** involves a number line as part of synthesizing fraction and decimal operations and reasoning.

In earlier grades, task **3:6 Unit Fraction Ideas** is a conceptual task about this foundational fraction concept.

Answer (continued)

(3) Explanations may vary but could involve the idea that $\frac{1}{5}$ is smaller than $\frac{1}{4}$, so that $1 - \frac{4}{5}$ is less far to the left of 1 than $1 + \frac{1}{4}$ is to the right of 1. Other explanations could involve finding equivalent fractions, including equivalent fractions with denominator 100. Diagrams may also vary; see the example.



Click here for a student-facing version of the task.

Refer to the Standards

4.NF.A; MP.3, MP.6, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

Students create a wide variety of mathematical products in task 4:4 by generating equivalent fractions, making statements of comparison, writing or speaking mathematical reasoning and conclusions, and creating mathematical representations.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:4? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:4? In what specific ways do they differ from 4:4?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† This understanding in turn is based on perhaps the single most foundational fraction concept, that of the unit fraction (for more on this concept see the Teacher Notes for task **3:6 Unit Fraction Ideas**).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?