4:6 Jar of Pennies

Teacher Notes

Math Milestones™

Central math concepts

Whereas the question 8,742 ÷ 5 = ? poses a procedural task, the question "How many nickels did Grandpa get?" poses a modeling task. That's because significant interpretation is required to relate the mathematics chosen by the student to the situation at hand. An equation model for Grandpa's situation might read as follows:

$$8{,}742 = 5 \times 1{,}748 + 2 \qquad (*)$$

Each number in this equation has a meaning in the problem context: 8,742 is the value of Grandpa's pennies, in cents; 5 is the value of a nickel, in cents; 1,748 is the number of nickels Grandpa will get; and 2 is the number of pennies due back to Grandpa from the teller. Procedural skill is involved in determining the numbers 1,748 and 2, given the numbers 8,742 and 5; modeling skill is involved in interpreting those numbers in context.

The number 1,748 in Equation (*) is often called the "quotient" of 8,742 and 5, but in more precise language 1,748 would be called the *whole-number quotient* of 8,742 and 5. The quotient of 8,742 and 5 is not 1,748; it's $1{,}748\,\frac{2}{5}$. To see this, note that

$$\begin{aligned} 5 \times \left(1{,}748\,\tfrac{2}{5}\right) &= 5 \times \left(1{,}748 + \tfrac{2}{5}\right) \\ &= 5 \times 1{,}748 + 5 \times \tfrac{2}{5} \quad \text{(using the distributive property)} \\ &= 8{,}740 + 2 \\ &= 8{,}742 \end{aligned}$$

In other words, $1{,}748\,\frac{2}{5}$ is the number which, when multiplied by 5, yields 8,742. This shows that $8{,}742 \div 5 = 1{,}748\,\frac{2}{5}$, because $C \div A$ is the unknown factor in $A \times ? = C$. This relationship is a trustworthy guide to mathematical reasoning, used frequently beginning in grade 3 and continuing at least until grade 7. The relationship recurs in high school as well, with the study of rational expressions, rational functions, and complex numbers. Consistent with so fundamental and useful a principle, a student should always be able to check a quotient by multiplying.

Another possible equation model for Grandpa's situation might be:

$$(8{,}742 - 2) \div 5 = 1{,}748 \qquad (**)$$

This equation tells its own story about the situation, a story in which we could imagine Grandpa removing two pennies from the jar before handing it over to the bank teller. Then the value of the remaining pennies would equal the value of a whole number of nickels. (Maybe Grandpa made a lucky guess here!)

We can imagine partitioning a cup of flour into smaller and smaller quantities of flour, but we don't think that way about discrete quantities like lunch trays, marbles, subway seats, and school crossing guards. So in a word problem, the context determines the best answer to give—whether this be the exact quotient of two given numbers, the whole number quotient, the quotient rounded to the next greatest whole number, or even the remainder.

4:6 Grandpa took a jar of pennies to the bank. He said, "I'd like nickels for this, please." The bank teller poured the pennies into a counting machine. "Eighty-seven dollars and forty-two cents," said the teller. **(1)** How many nickels did Grandpa get? **(2)** Check your answer with an estimate.

Answer

(1) 1,748. **(2)** Estimates will vary. For example, an estimate of 2,000 could come from rounding \$87.42 to \$100, then reasoning that since \$1 is worth 20 nickels, \$100 is worth 2,000 nickels.

Click here for a student-facing version of the task.

Refer to the Standards

4.OA.A.3; MP.1, MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application, Procedural skill and fluency

Additional notes on the design of the task

- In the text of task 4:6, words that suggest division are absent. This is intended is to disincentivize "key word hunting" and also make room for equation models like the one in Equation (*) that involve multiplication and addition.
- Because of the lack of key words, it might be useful for students to act out the problem, one playing the part of Grandpa and another playing the part of the bank teller.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental calculation; finding quotients of multi-digit numbers; rewriting decimal quantities as whole number multiples of a smaller unit; and rounding and estimation.

Extending the task

How might students drive the conversation further?

- Students could recommend that Grandpa look in his pockets for three more pennies.
- Students could judge that 1,748 nickels is an inconveniently large number of nickels and ask what Grandpa would have received instead if he had asked for quarters.
- Students might know that banks sometimes handle rolls of coins. A roll of nickels has a value of $2. How many nickels is that? If Grandpa had asked for the nickels in rolls, how many rolls would that be?

Related Math Milestones tasks

4:11

4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

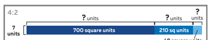
4:10

4:10 Write the values of the products and quotients. Check the quotients by multiplying.

Mentally: 40×20, 30×11, 12×60, 5×19, $480 \div 8$

With pencil and paper: $6{,}132 \times 6$, 48×39, $7\overline{)8{,}722}$

4:2

4:2


(1) Find the three missing lengths and write them on the diagram. Compare answers with a classmate.

(2) What is the total area of the diagram?

(3) Look for connections between the diagram and the division problem. What connections can you see?

$7\overline{)959}$ = 137; 959 − 700 = 259; 259 − 210 = 49; 49 − 49 = 0

Task **4:11 School Kitchen** involves division in context, with the added element that the given quantities have different units. Task **4:10 Calculating Products and Quotients** involves multi-digit procedures like those which could be used for task 4:6. Task **4:2 Multi-Digit Division Concepts** involves the standard multi-digit division algorithm with a conceptual focus.

5:2

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

In later grades, the progression of dividing with whole numbers comes to a close as the fraction $\frac{A}{B}$ becomes understood as the quotient $A \div B$; task **5:2 Water Relief** centers on this concept in context, and task **5:5 Calculating** includes problems involving division of whole numbers leading to a non-whole-number quotient.

3:1

3:1 There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: ____________

3:4

3:4 Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: ____________

Answer: ____________

3:9

3:9 Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

In earlier grades, tasks **3:1 Volleyball Players**, **3:4 Corn Seeds**, and **3:9 Bulletin Board Pictures** form a kind of survey of the essential early meanings of multiplication and division; the requested equation models can support learning about the relationship between the operations.

Additional notes on the design of the task (continued)

- The artwork consisting of a collection of 5 pennies could be useful for prompting mathematical thinking about how to approach the problem. Students who don't know that nickels are worth 5 cents can be given this information.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:6? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:6? In what specific ways do they differ from 4:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?