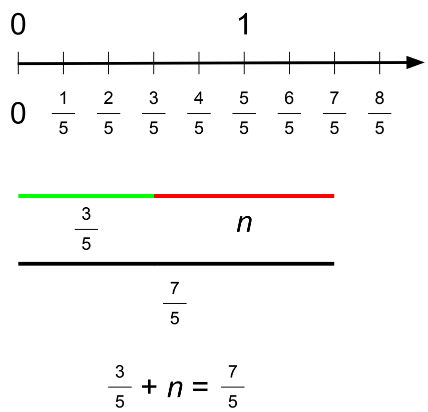
4:7 Fraction Sums and Differences

Teacher Notes

Math Milestones™

Central math concepts

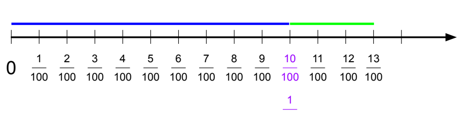
Extending addition and subtraction from whole numbers to fractions enables students to apply mathematics not only in relation to discrete objects, like bowling balls or boats, but also in relation to substances that can be measured out and repeatedly subdivided, like a quantity of fluid or an interval of time. Often, the measures of these substances are fractions. As students extend addition and subtraction from whole numbers to fractions, the meanings of the operations remain the same: adding remains an operation of joining or adding-to, and subtraction remains an operation of separating or taking-from. Also unchanged, then, is the relationship between addition and subtraction: just as with whole numbers, $C - A$ is the unknown addend in $A + \square = C$. Because a number line shows the continuum of all numbers, a number line can efficiently represent sums and differences of fractions (see figure, which shows the unknown addend problem $\frac{3}{5} + n = \frac{7}{5}$).



Unit fractions $\frac{1}{b}$ are the building blocks of fractions,† in the multiplicative sense that a general fraction $\frac{a}{b}$ is a copies of $\frac{1}{b}$ (written in symbols, $\frac{a}{b} = a \times \frac{1}{b}$), and also in the additive sense that in a problem like $\frac{3}{5} + \frac{2}{5} = ?$, the unit fraction $\frac{1}{5}$ can be thought of as a unit (one "fifth"), so that the sum $\frac{2}{5} + \frac{3}{5}$ can be thought of as 2 of the unit plus 3 of the unit, yielding 5 of the unit (5 fifths or $\frac{5}{5}$). Thanks to unit fractions, the unknown-addend problem shown in the figure, $\frac{3}{5} + n = \frac{7}{5}$, could be solved by thinking, "3 fifths plus how many fifths equals 7 fifths?" in somewhat the same way that students in first grade might have thought about the problem "3 eggs plus how many eggs is 7 eggs?"

Conceptual understanding of fraction addition and subtraction with equal denominators will support students in adding and subtracting fractions with unequal denominators in grade 5, because the core strategy for adding and subtracting with unequal denominators is to use the principle of fraction equivalence to replace the given problem with a problem in which the denominators are equal.

This strategy also applies to one of the sums in task 4:7, namely $\frac{1}{10} + \frac{3}{100}$. Using the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$, the first addend $\frac{1}{10}$ is equal to $\frac{10 \times 1}{10 \times 10} = \frac{10}{100}$. So the sum $\frac{1}{10} + \frac{3}{100}$ equals $\frac{10}{100} + \frac{3}{100}$ (see the figure).



4:7 Write the values of the expressions. Read each completed equation aloud.

3 fifths + 2 fifths = ___

$\frac{1}{10} + \frac{3}{100} =$ ___ (fraction)

= ___ (decimal)

$\frac{6}{25} + \frac{6}{25} =$ ___

$\frac{1}{8} + \frac{5}{8} - \frac{3}{8} =$ ___

Answer

5 fifths or $\frac{5}{5}$ or 1; $\frac{13}{100}$, 0.13; $\frac{12}{25}$; $\frac{3}{8}$.

Click here for a student-facing version of the task.

Refer to the Standards

4.NF.B.3a–c, 4.NF.C.5, 6; MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- Task 4:7 is designed to target conceptual understanding, even though it only asks for brief answers rather than asking for extended writing or making other language demands. Teachers can also question students about the thinking that led to their answers, individually or in a group setting (and students can question each other).

In terms of unit thinking, the problem $\frac{1}{10} + \frac{3}{100} = ?$ is like asking for the total value of 1 dime and 3 pennies, a problem we could solve by using a common unit of pennies for both values.

Task 4:7 also asks for the total $\frac{13}{100}$ in decimal notation as 0.13. Using decimal notation for fractions that are multiples of $\frac{1}{10}$ or multiples of $\frac{1}{100}$ is the opening for the full study of the place value system in grade 5.

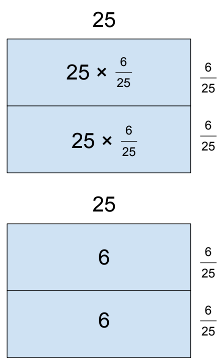
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: working with unit fractions; single-digit sums and differences; and understanding the meanings of the operations of addition and subtraction.

Extending the task

How might students drive the conversation further?

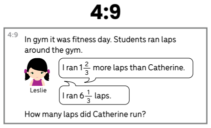
- Students could analyze an area model to understand that $25 \times (\frac{6}{25} + \frac{6}{25}) = 25 \times \frac{6}{25} + 25 \times \frac{6}{25}$. And since each addend is 6, the sum is 12; in other words, the result of $25 \times (\frac{6}{25} + \frac{6}{25})$ should be 12. Students can then check: is 12 the result you get from multiplying their answer to $\frac{6}{25} + \frac{6}{25} = ?$ by 12?
- Students could produce the area model and apply the reasoning to another case that they produce, like $7 \times (\frac{2}{7} + \frac{2}{7}) = 7 \times \frac{2}{7} + 7 \times \frac{2}{7} = 2 + 2 = 4$.



Not to scale

Related Math Milestones tasks

4:9

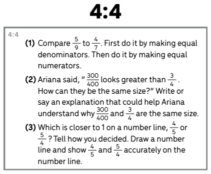

4:9 In gym it was fitness day. Students ran laps around the gym.

Leslie: I ran $1\frac{2}{3}$ more laps than Catherine.

I ran $6\frac{1}{3}$ laps.

How many laps did Catherine run?

4:4


4:4

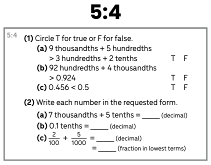
(1) Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.

(2) Ariana said, "$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?" Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.

(3) Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

Task **4:9 Fitness Day** is a word problem involving addition and subtraction with fractions. Task **4:4 Comparing Fractions with Equivalence** involves the principle of fraction equivalence in connection with fraction size.

5:4


5:4 **(1)** Circle T for true or F for false.

(a) 9 thousandths + 5 hundredths > 3 hundredths + 2 tenths T F

(b) 92 hundredths + 4 thousandths > 0.924 T F

(c) 0.456 < 0.5 T F

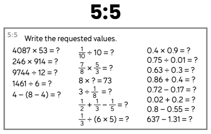
(2) Write each number in the requested form.

(a) 7 thousandths + 5 tenths = _____ (decimal)

(b) 0.1 tenths = _____ (decimal)

(c) $\frac{2}{100} + \frac{5}{1000}$ = _____ (decimal) = _____ (fraction in lowest terms)

5:5

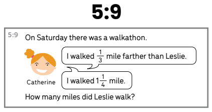

5:5 Write the requested values.

$4087 \times 53 = ?$ $246 \times 914 = ?$ $9744 \div 12 = ?$ $1461 \div 6 = ?$ $4 - (8 - 4) = ?$

$\frac{1}{10} \div 10 = ?$ $\frac{7}{8} \times \frac{5}{3} = ?$ $8 \times ? = 73$ $3 \div \frac{1}{8} = ?$ $\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$ $\frac{1}{3} \div (6 \times 5) = ?$

$0.4 \times 0.9 = ?$ $0.75 \div 0.01 = ?$ $0.63 \div 0.3 = ?$ $0.86 + 0.4 = ?$ $0.72 - 0.17 = ?$ $0.02 + 0.2 = ?$ $0.8 - 0.55 = ?$ $637 - 1.31 = ?$

5:9

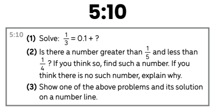

5:9 On Saturday there was a walkathon.

Catherine: I walked $\frac{1}{3}$ mile farther than Leslie.

I walked $1\frac{1}{4}$ mile.

How many miles did Leslie walk?

5:10


5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$

(2) Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.

(3) Show one of the above problems and its solution on a number line.

In later grades, task **5:4 Place Value to Thousandths** situates problems like $\frac{2}{10} + \frac{5}{1000}$ in the context of decimal place value. Task **5:5 Calculating**

Additional notes on the design of the task (continued)

- "Read each completed equation aloud" is intended to leverage the way spoken names of fractions evoke units, as in "Six twenty-fifths plus six twenty-fifths equals twelve twenty-fifths." And in this case, the use of two sixes is intended to evoke a potentially familiar "doubles fact" from primary-grades addition work.

Curriculum connection

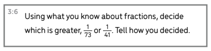
1. In which unit of your curriculum would you expect to find tasks like 4:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:7? In what specific ways do they differ from 4:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See Zimba (2013), "Units, a Unifying Idea in Measurement, Fractions, and Base Ten."

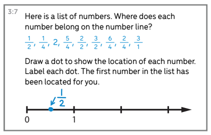
* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

involves sums and differences of fractions with unequal denominators from a procedural point of view, while task **5:9 Walkathon** involves such a calculation in context. Task **5:10 Number System, Number Line** part (1) is an unknown-addend problem that includes both a fraction and a decimal.

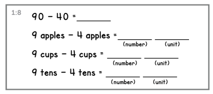
3:6



3:7



1:8



In earlier grades, task **3:6 Unit Fraction Ideas** focuses on these building blocks of fractions, and task **3:7 Locating Numbers on a Number Line** involves whole numbers and fractions together on the number line including simple cases of equivalent fractions. Task **1:8 Subtracting Units** involves thinking about subtraction of whole numbers in units of tens, which is analogous to thinking about subtraction of fractions in terms of unit fractions.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?