# 4:8 Shapes with Given Positions

Teacher Notes

## Central math concepts

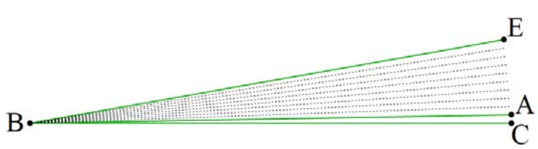
A geometric point with zero size is an abstraction, since any physical drawing of a point, no matter how sharp the pencil we use to draw it, has nonzero diameter. Therefore a point is an idea, and understanding that idea necessarily involves the imagination. Similarly, a geometric line or ray is infinite in extent, even though no physical drawing of a line could be infinitely long or perfectly straight. And the plane to which all these figures belong is no less imaginary an object, given its perfect flatness and its lack of edges. Indeed the infinite sizes of lines and planes, and their constitution as infinitely dense sets of points, are what make them suitable as *number* lines and *coordinate* planes—since the real numbers themselves are both infinite and infinitely dense.

Fortunately, it isn't necessary to draw infinitely long or infinitely straight lines in order to reason about them. Rather, geometric reasoning proceeds on the basis of creating, analyzing, and discussing diagrams that depict geometric objects and relationships. These diagrams have to be true enough for the purpose, but they will usually have conventional features that aren't to be taken literally, such as the thickness of the red point of origin of ray *R* in task 4:8, or the red arrowheads on line *L*. (Lines don't have arrowheads, nor do they have endpoints to which an arrowhead could be attached.) New learners may need to discuss the ways in which a diagram does and doesn't depict geometric "reality."

Geometric measurement advances in grade 4 with angle measurement. Angle measurement resembles other kinds of measurement in its reliance on a unit (CCSS 4.MD.C.5):

- An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.
- An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

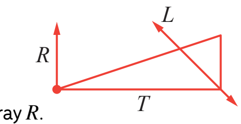
For example, the figure shows a one-degree angle ABC being used as a unit to measure a ten-degree angle EBC.




As shown in the table, ideas of angle measure are parallel with ideas of area measure from grade 3, when students learn to recognize area as a measurable attribute of plane figures and to understand concepts of area measurement (CCSS 3.MD.C.5).

4:8 *L* is a line, *R* is a ray, and *T* is a triangle.
True or false:

**(1)** Line *L* is a line of symmetry for triangle *T*.




**(2)** Line *L* intersects ray *R*.

**(3)** Triangle *T* has two angles measuring less than 90 degrees.

### Answer

**(1)** False. **(2)** True. **(b)** True.

Click here for a student-facing version of the task.

### Refer to the Standards

4.MD.C, 4.G.A; MP.5, MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

### Aspect(s) of rigor:

Concepts

### Additional notes on the design of the task

- Students can choose whether to use a protractor to answer part (3).
- The given information that "*L* is a line" and "*R* is a ray" means that these objects satisfy their geometric definitions; in particular, they aren't finite in length.

### Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 4:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 4:8? In what specific ways do they differ from 4:8?

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| Area Measure | Angle Measure |
|---|---|
| A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. | An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles. |
| A plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. | An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. |

Volume measurement provides additional parallels to angle, area, and length measurement; see CCSS 5.MD.C.3 and the Teacher Notes for task **5:3 Neighborhood Garden**.

## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using visual and spatial reasoning; and making measurements on a diagram.

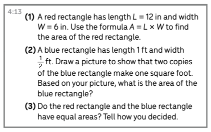
## Extending the task

How might students drive the conversation further?

- Students could trace the figure on a blank sheet of paper and use a straightedge to extend line *L* and ray *R* showing their intersection point.
- Students could discuss whether triangle *T* has any line of symmetry.

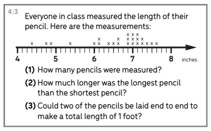
## Related Math Milestones tasks

**4:13**



4:13
(1) A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.
(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?
(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

**4:3**



4:3
Everyone in class measured the length of their pencil. Here are the measurements:

(1) How many pencils were measured?
(2) How much longer was the longest pencil than the shortest pencil?
(3) Could two of the pencils be laid end to end to make a total length of 1 foot?

Task **4:13 Area Units** involves geometric measurement in relation to area units of differing sizes. The arrowhead on the number line in task **4:3 Pencil Data** has similar purpose to the arrowheads on line *L* and ray *R* in task 4:8.

### Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 4:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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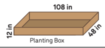
**5:8**

5:8 A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths. **(1)** Draw an example of an alana-gon. **(2)** True or false: **(a)** All squares are alana-gons. **(b)** No trapezoids are alana-gons.

**5:3**

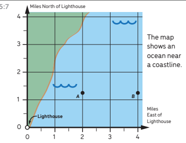
5:3 A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is 54 $ft^3$ of soil. How many truckloads of soil will fill all of the boxes?




Planting Box

**5:7**

5:7




The map shows an ocean near a coastline.

Shipwrecks are at locations *A* $(2, 1\frac{1}{4})$ and *B* $(4, 1\frac{1}{4})$. Shipwrecks are also at locations *C* $(4, 3\frac{1}{2})$ and *D* $(2, 3\frac{1}{2})$. **(1)** Mark *C* and *D* on the map and shade rectangle *ABCD*. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in $mi^2$?

**5:11**

5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
**(1)** Is Juliet thinking of something possible or impossible? Use math to decide for sure.
**(2)** Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

**6:12**

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

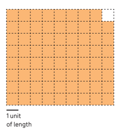
In later grades, task **5:8 Alana's New Shape Category** involves classification and hierarchy of shapes. Tasks **5:3 Neighborhood Garden**, **5:7 Shipwrecks**, and **5:11 Juliet's Rectangle** involve geometric measurement. Task **6:12 Coordinate Triangle** places a geometric figure in the coordinate plane.

**3:8**

3:8
**(1)** Name two attributes that are shared by triangles and squares.
**(2)** Name a category of shapes that includes triangles and squares and also includes other shapes that have both of the attributes you named.

**3:3**

3:3
**(1)** How much area is shaded?




**(2)** Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ____________ Width: ____________

In earlier grades, task **3:8 Shape Attributes and Categories** involves defining attributes and classification of shapes. Task **:3 Length and Area Quantities** involves concepts of area measurement.

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## Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

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