5:10 Number System, Number Line

Teacher Notes

Math Milestones™

Central math concepts

In mathematics and science, two powerful conceptions of number live side by side. In one conception, a real number is a label for a point on the number line. And in a related but distinct conception, a real number is a possible magnitude of some quantity measured with a chosen unit. Both conceptions of number are learned as early as grade 3.[†] Importantly, in neither conception is the *format* of the number essential: we can label points on the number line with fractions like $\frac{3}{4}$, or we can label them with multi-digit numbers like 0.75. And we can express a length measurement as $\frac{1}{5}$ mile, or we can express it as 0.2 mile.

Just as numbers can be understood without reference to their format, so can operations be understood independently of the format of the numbers on which they operate. For example, subtraction is an operation of separating or taking-from, regardless of whether the quantities being separated or decreased are written as fractions or as multi-digit symbols using base-ten.

Calculation algorithms, on the other hand, do depend on a number's format. The algorithms for fractions and multi-digit numbers are very different, yet the core meanings of division (quotative and partitive) are the same regardless of the format of the numbers involved.

> 6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Task **6:14 Dividing Decimals and Fractions**. The division operation has the same meaning in parts (1) and (2), but the calculation algorithm depends on the number format in each case.

Because calculation algorithms differ so greatly depending on the format of the numbers, curriculum lessons and units often concentrate separately on fractions and decimals. But too hard a wall of separation risks obscuring fundamental concepts about both numbers and operations. In part (1) of task 5:10, the unknown number is $0.1 - \frac{1}{3}$ because $\frac{1}{3} = 0.1 + ?$ is an unknown addend problem. The idea of subtraction as an unknown addend problem is an idea so elementary that kindergarten students work with it, and so profound that students will continue to rely on it throughout their work with fractions and decimals, rational numbers, real numbers, algebraic expressions, complex numbers, and matrices.

Equally profound are the concepts of number that are conveyed by the metaphor of the number line. As a geometric line, the number line is infinitely dense with points, meaning that between any two points there are infinitely many points; therefore, given any two numbers, there are infinitely many numbers with values in between. The measurement conception of a number agrees with that idea too, because given any unit, no matter how small, one can always imagine partitioning it and using one of the parts as a new, smaller unit. This smaller unit can then be iterated on the number line with no gaps or overlaps. For example, with respect to part (3) of task 5:10, we could iterate the unit $\frac{1}{40}$ on the number

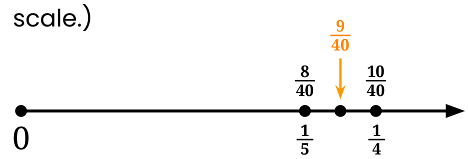
> 5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$
> **(2)** Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.
> **(3)** Show one of the above problems and its solution on a number line.

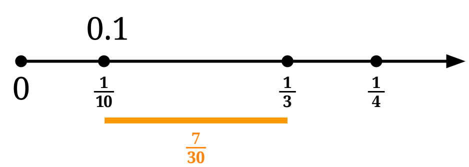
Answer

(1) $\frac{7}{30}$. (Note: the repeating decimal answer $0.2\overline{3}$ isn't necessarily expected at this grade, but it is also correct.)

(2) There is a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$. Examples may vary: for example, $\frac{9}{40}$, $\frac{2}{9}$, $\frac{21}{100} = 0.21$, $\frac{22}{100} = \frac{11}{50} = 0.22$, $\frac{23}{100} = 0.23$, $\frac{24}{100} = \frac{6}{25} = 0.24$, $\frac{1}{5} + \frac{1}{1{,}000{,}000} = 0.200001$, $\frac{1}{4} - \frac{1}{1{,}000{,}000} = 0.249999$.

(3) See the figures for two possibilities. (Note: each figure is drawn approximately to scale, but the two figures are not drawn to the same scale.)





Click here for a student-facing version of the task.

Refer to the Standards

5.NF.A.1; MP.1, MP.3, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

line and thereby represent the magnitudes $\frac{8}{40}$, $\frac{9}{40}$, $\frac{10}{40}$, in particular representing a magnitude $\frac{9}{40}$ that is intermediate between $\frac{8}{40} = \frac{1}{5}$ and $\frac{10}{40} = \frac{1}{4}$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: renaming fractions with denominators of 10 to decimals; understanding subtraction as an unknown addend problem; generating equivalent fractions; number sense of fraction sizes; and using a number line to support and communicate mathematical reasoning.

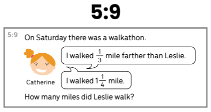
Extending the task

How might students drive the conversation further?

- Finding a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$ might lead students to wonder whether it is the case that between any two different numbers there is always another number. Exploring this question could lead to a more general insight: between any two different numbers there are always infinitely many other numbers.
- Students could make sense of parts (1) and (2) by creating a word problem for each. For example, “A. ran $\frac{1}{4}$ mile. B. ran $\frac{1}{5}$ mile. I didn’t run as far as A., but I ran farther than B. What is a distance I could have run?”

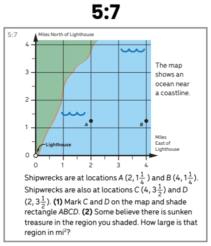
Related Math Milestones tasks

5:9


5:9 On Saturday there was a walkathon.

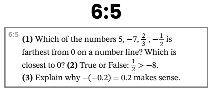
How many miles did Leslie walk?

5:7

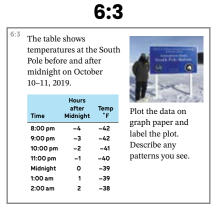

5:7

Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

6:5


6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

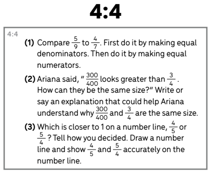
6:3


6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

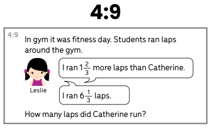
Time	Hours after Midnight	Temp °F
8:00 pm	–4	–42
9:00 pm	–3	–42
10:00 pm	–2	–41
11:00 pm	–1	–40
Midnight	0	–39
1:00 am	1	–39
2:00 am	2	–38

Plot the data on graph paper and label the plot. Describe any patterns you see.

4:4


4:4 **(1)** Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.
(2) Ariana said, “$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?” Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.
(3) Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

4:9


4:9 In gym it was fitness day. Students ran laps around the gym.

How many laps did Catherine run?

Task **5:9 Walkathon** is a word problem for which the equation model could be an unknown-addend problem with a fraction and a mixed number. Task **5:7 Shipwrecks** involves the extension of the number line to the coordinate plane.

In later grades, task **6:5 Positive and Negative Numbers** emphasizes understanding a rational number as a point on the number line, and task **6:3 South Pole Temperatures** involves using the coordinate plane to plot a data set with observations that include signed numbers.

In earlier grades, task **4:4 Comparing Fractions with Equivalence** involves ideas of fraction magnitude and the number line. Task **4:9 Fitness Day** is a word problem for which the equation model could be an unknown-addend problem with mixed numbers (equal denominators).

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

The task rewards persistence, in the sense that parts (1) and (2) both present initial challenges. In part (1), the fraction $\frac{1}{3}$ cannot be written as a terminating decimal. In part (2), it is natural to begin by finding the least common denominator of $\frac{1}{4}$ and $\frac{1}{5}$, but that choice of denominator results in consecutive numerators (5 and 4).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:10? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:10? In what specific ways do they differ from 5:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit*

† See domain 3.NF in the Standards, when students ‘understand a fraction as a number on the number line’ and also ‘define the interval from 0 to 1 as the whole and partition it into b equal parts,’ ‘recognizing that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.’ (Single quotation marks indicate paraphrase; see the exact language online.)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?