5:11 Juliet's Rectangle

Teacher Notes

Math Milestones™

Central math concepts

If you ask someone to draw a rectangle, they will probably draw a rectangle that is just a little wider than it is tall. However, rectangles can be any shape that satisfies the mathematical definition. True, it would probably be impossible to draw Juliet's rectangle using physical length units; for example, if the length unit were 1 foot, then Juliet's rectangle would be no wider than a bacterium, and over 90 miles long! Nevertheless, mathematical rectangles exist with any positive numbers as dimensions, and we may think about these rectangles and draw mathematical conclusions about them.

One mathematical conclusion we might draw about Juliet's rectangle is that the side-length measurements can't both be whole numbers. That's because the only whole-number factors with a product of 1 are the factors 1 and 1, but the perimeter of a 1-by-1 rectangle is 4 units, too small to meet Juliet's perimeter condition. Thus, Juliet's rectangle takes us beyond whole numbers, potentially raising questions about the meaning of multiplying by a fraction.

In general, the product $(\frac{a}{b}) \times q$ is a parts of a partition of q into b equal parts, equivalently the result of a sequence of operations $a \times q \div b$ (CCSS 5.NF.B.4). So for example, in Juliet's rectangle, suppose we take the length to be 500,000 units, which will certainly satisfy the perimeter requirement. Because the rectangle has area 1 square unit, the width of the rectangle is the unknown factor in ? × 500,000 = 1. Therefore, the unknown factor is 1 ÷ 500,000, and that quantity is 1 part in a partition of 1 into 500,000 equal parts, or $\frac{1}{500{,}000}$. To check this quotient, we can multiply. Doing so, $\frac{1}{500{,}000}$ × 500,000 is 1 part in a partition of 500,000 into 500,000 parts, and that's 1. Alternatively, $\frac{1}{500{,}000}$ × 500,000 is the result of the sequence of operations 1 × 500,000 ÷ 500,000, which is 1.

In Juliet's rectangle, the side-length measurements L and W satisfy the condition that $L \times W = 1$. This makes the numbers L and W an illustration of an important property of operations, the existence of multiplicative inverses. Whatever nonzero value W has, L is its multiplicative inverse, and vice versa.

Extending multiplication and division from whole numbers to fractions is perhaps the most important mathematical progression of the upper-elementary grades. This progression involves a substantial evolution in students' concepts about numbers and operations. The need to support students in making that conceptual evolution raises important questions: What aspects of earlier thinking about multiplying whole numbers will remain helpful when making sense of a product involving fractions? What new ways of thinking will be helpful? And what mathematical representations introduced during whole-number multiplication work are best suited to supporting the transition from multiplying and dividing whole numbers to multiplying and dividing all numbers?

5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."

(1) Is Juliet thinking of something possible or impossible? Use math to decide for sure.

(2) Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

Answer

(1) It is possible for a rectangle to have an area of 1 square unit and a perimeter of more than 1 million units. For example, the rectangle might have length $\frac{1}{1{,}000{,}000}$ unit and width 1,000,000 units. Then the area is 1 square unit, and the perimeter is 2,000,000 + $\frac{2}{1{,}000{,}000}$ units. A distinct kind of argument could establish logically that Juliet's rectangle is possible without actually constructing a specific example of one; for example, it could be argued that "If we make the length be 1 divided by the width, then the width could be very large, like over 1 million units, but the area would still be 1 square unit." **(2)** Answers may vary but should include a noticeable revision toward such improvements as more complete and precise language, adding a labeled diagram, redrawing a diagram for greater clarity, etc.

Click here for a student-facing version of the task.

Refer to the Standards

5.NF.B; MP.1–3, MP.6, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using large whole numbers; thinking about length and area units; working with unit fractions; multiplying by a unit fraction; and basing multiplicative reasoning on math diagrams.

Extending the task

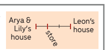
How might students drive the conversation further?

- Students could generate several different rectangles that satisfy Juliet's area and perimeter conditions and then look for patterns. For example, is 500,000 units the smallest possible length that Juliet's rectangle could have? Is there a largest possible length that Juliet's rectangle could have?
- Students could make up their own versions of Juliet's rectangle, such as, "I am thinking of a rectangle with area 12 square units and perimeter greater than 24 units. What might be the dimensions of my rectangle?"

Related Math Milestones tasks

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

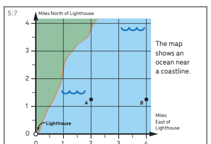


5:14

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

5:7

5:7



The map shows an ocean near a coastline.

Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

Fraction products receive a conceptual emphasis in tasks **5:6 Corner Store** and **5:14 Brandon's Equation**, an application emphasis in task **5:7 Shipwrecks**, and a procedural emphasis in task **5:5 Calculating**.

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

In later grades, tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** are unknown-factor problems involving fractions. Task **6:13 Is There a Solution? (Multiplication)** is like Juliet's Rectangle in that the product of a whole number with an unknown number results in a product less than the whole number.

Additional notes on the design of the task

Note that the perimeter of the rectangle is only required to be "more than 1 million units"; the perimeter doesn't have to *equal* 1 million units.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:11? In what specific ways do they differ from 5:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

4:13

4:13

(1) A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

4:5

4:5

(1a–f) Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)

$6 \times \frac{4}{7} =$ ___ (b)

$86 \times \frac{1}{86} =$ ___ (c)

$6 \times \frac{8}{2} =$ ___ (d)

$9 \times \frac{1}{9} =$ ___ (e)

$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

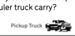
(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

4:12

4:12

The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?



4:8

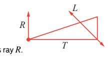
4:8

L is a line, R is a ray, and T is a triangle. True or false:

(1) Line L is a line of symmetry for triangle T.

(2) Line L intersects ray R.

(3) Triangle T has two angles measuring less than 90 degrees.



In earlier grades, task **4:13 Area Units** involves rectangle area in a case of a fractional length. Task **4:5 Fraction Products and Properties** involves multiplying a fraction by a whole number with a conceptual emphasis, and task **4:12 Super Hauler Truck** involves a multiplicative comparison leading to multiplying a fraction by a whole number in context. Task **4:8 Shapes with Given Positions** involves the distinction between a geometric (mathematical) object and the physical diagram that refers to it.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?