5:13 Frozen Yogurt Machine

Teacher Notes

Math Milestones™

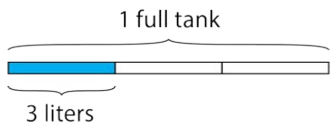
Central math concepts

Step 1. Task 5:13 is typical of multi-step word problems in that a quantity worth knowing in the situation isn't named in the text of the problem. That quantity is the number of liters in the machine when it is full. The value of this quantity isn't given—it also isn't asked for—yet the value is helpful for solving the problem that is posed. Finding this value involves thinking about unit fractions and division.

As students extend their understanding of multiplication and division from whole numbers to fractions, one idea that endures is the idea that division finds an unknown factor: $C \div A$ is the unknown factor in $A \times ? = C$. In task 5:13, the number of liters in the machine when it is full is initially an unknown factor, because $\frac{1}{3}$ of this number is 3:

$\frac{1}{3} \times ? = 3.$

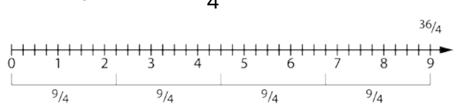
The unknown factor is 9, so that $3 \div \frac{1}{3} = 9$. See the figure for a representation of the unknown factor problem $\frac{1}{3} \times ? = 3$.



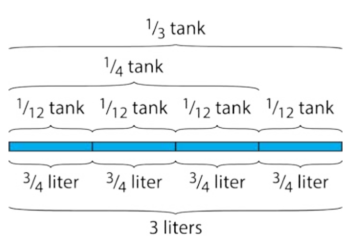
The figure also suggests more generally why dividing by a unit fraction $\frac{1}{n}$ amounts to multiplying by its reciprocal, n. For an illuminating discussion of fraction division with numerous diagrams and examples, see the 2017 series of blog posts by William McCallum and Kristin Umland on MathematicalMusings.org (part 1, part 2, part 3, part 4). In particular, part 3 concentrates on the reason why "dividing by a unit fraction is the same as multiplying by its reciprocal."

Step 2. Knowing there are 9 liters in the machine when it is full, the quantity of yogurt in the machine, in liters, when the machine is $\frac{1}{4}$ full is given by the product $\frac{1}{4} \times 9$. One-fourth of 9 means 1 part of a partition of 9 into 4 equal parts, or equivalently $9 \div 4$, and this quotient is $\frac{9}{4}$.

Partitioning 9 into 4 equal parts could also be accomplished by thinking about units and fraction equivalence, as shown in the diagram. First rewrite the number 9 as a fraction, $\frac{36}{4}$. This fraction is 36 parts of size one-fourth. When 36 parts of size one-fourth are partitioned into 4 equal parts, 1 of the parts will consist of 9 parts of size one-fourth; and 9 parts of size one-fourth are $\frac{9}{4}$.



Another way. (See the diagram.) When the yogurt in the machine decreases from $\frac{1}{3}$ of a tank to $\frac{1}{4}$ of a tank, it decreases by $\frac{1}{12}$ of a tank (because $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$). That $\frac{1}{12}$ of a tank is $\frac{1}{4}$ of the 3 liters, because $\frac{1}{12}$ is $\frac{1}{4}$ of $\frac{1}{3}$. And $\frac{1}{4}$ of 3 liters is $\frac{3}{4}$ ℓ. So when the yogurt in the machine decreases from $\frac{1}{3}$ full to $\frac{1}{4}$ full, it decreases by $\frac{3}{4}$ ℓ of yogurt. This gives the answer to the problem as $3 - \frac{3}{4} = 2\frac{1}{4}$ liters.



5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Answer

$\frac{9}{4}$ ℓ, $2\frac{1}{4}$ ℓ, or 2.25 ℓ.

Click here for a student-facing version of the task.

Refer to the Standards

5.NF.B.6, 7; MP.1, MP.2, MP.4, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

One approach to the problem would involve division of a whole number by a unit fraction, followed by multiplication of a whole number by a unit fraction; this approach could be expressed by the calculation $\frac{1}{4} \times (3 \div \frac{1}{3})$. However, students' thought processes might correspond instead to equivalent expressions, such as $(3 \times 3) \div 4$.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:13? In what specific ways do they differ from 5:13?

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying by a unit fraction; applying ideas of times-as-much in context; and basing multiplicative reasoning on math diagrams.

Extending the task

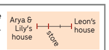
How might students drive the conversation further?

- Knowing that the machine holds $2\frac{1}{4}$ ℓ of frozen yogurt when it is $\frac{1}{4}$ full, students could observe that decreasing from $\frac{1}{3}$ full to $\frac{1}{4}$ full corresponded to a decrease of $3 - 2\frac{1}{4} = \frac{3}{4}$ ℓ of frozen yogurt. What fraction of a full tank is $\frac{3}{4}$ ℓ ? How can that fraction be seen in the situation?
- A student who approached the problem in one way (or drew one kind of diagram) could present their thinking to a partner who approached it a different way (or drew another kind of diagram). The partner could say back what the first student said. The two students could refine their explanations and responses until the mathematics has been effectively communicated.

Related Math Milestones tasks

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



5:2

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

5:11

5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
(1) Is Juliet thinking of something possible or impossible? Use math to decide for sure.
(2) Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

Task **5:6 Corner Store** involves fraction products in context. Task **5:2 Water Relief** involves interpreting the quotient of two whole numbers as a fraction. Task **5:11 Juliet's Rectangle** could promote thinking about the extension of multiplication from whole numbers to fractions.

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

In later grades, tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** are word problems that use multiplication and division operations on fractions, completing the extension of arithmetic from whole numbers to fractions. Task **6:6 Planting Corn** is about proportionality (which can involve application of times as much thinking). In task **6:13 Is There a Solution? (Multiplication)**, the equation can be made sense of as a question about multiplicative scaling.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

4:5

4:5

(1a–f) Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ____ (a)

$6 \times \frac{4}{7} =$ ____ (b)

$86 \times \frac{1}{86} =$ ____ (c)

$6 \times \frac{8}{2} =$ ____ (d)

$9 \times \frac{1}{9} =$ ____ (e)

$9 \times \frac{2}{9} =$ ____ (f)

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

4:1

4:1

A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

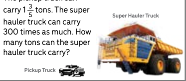
Equation model: ____________

Answer: ____________

4:12

4:12

The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?



3:6

3:6

Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

In earlier grades, task **4:5 Fraction Products and Properties** can involve times-as-much thinking and sense-making about fraction products. Tasks **4:1 Tablespoon of Oil** (whole numbers) and **4:12 Super Hauler Truck** (fractions) are word problems involving multiplicative comparison. The building blocks of fractions are the topic in task **3:6 Unit Fraction Ideas**.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?