5:1 Juice Box Mixup

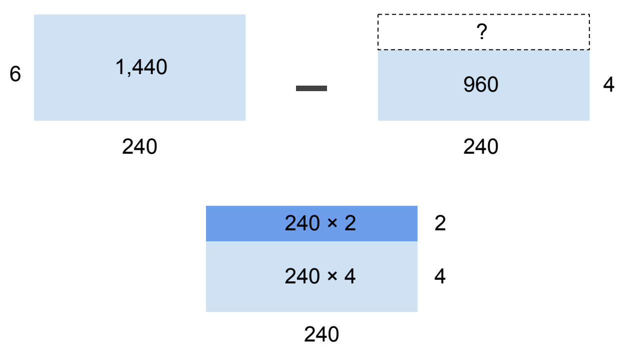
Teacher Notes

Central math concepts

Comparing different approaches to the same problem often reveals important mathematics that would be hard to see within just a single approach. To illustrate the central math concepts in this task, consider the following two distinct approaches to the task:

> *Approach 1*. First, calculate how many juice boxes the school bought: 240 × 6 = 1,440. Next, calculate how many juice boxes the school needed: 240 × 4 = 960. So the school bought 1,440 juice boxes, but the school only needed 960 juice boxes. Therefore, the school bought 1,440 - 960 = 480 extra juice boxes.
>
> *Approach 2*. Because the school only needed to buy 4-packs, every 6-pack bought by the school included 2 juice boxes the school didn't need. Therefore, the school bought 240 × 2 = 480 extra juice boxes.

Both ways of thinking about the problem are good and true, but then that raises a question: how can two such different ways of thinking lead to the same answer? There is a deep connection between the two approaches, and it involves the distributive property. In the figure, an area model is used to illustrate the connection:



These diagrams illustrate the central mathematical relationship between the two approaches, which is that (240 × 6) - (240 × 4) = 240 × 2. Said another way,

$$(240 \times 2) + (240 \times 4) = 240 \times 6.$$

And since 6 = 2 + 4, the previous equation can be written as

$$(240 \times 2) + (240 \times 4) = 240 \times (2 + 4),$$

which is a particular case of the distributive property, $a \times b + a \times c = a \times (b + c)$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying a multi-digit number by a single-digit number; using area models to illustrate reasoning that involves multiplication, addition or subtraction, and the

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

Answer

The school bought 480 extra juice boxes.

Click here for a student-facing version of the task.

Refer to the Standards

5.OA.A.2, 5.NBT.B.5; MP.4, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The task isn't intended to *differentiate between* students who adopt the strategy 240 × 2 = 480 and students who adopt the strategy 240 x 6 = 1,440; 240 x 4 = 960; 1,440 - 960 = 240. Rather, the task is designed to *foster mathematical conversations* that compare these or other approaches in relation to one another. In other words, the point isn't to decide which strategy is best, but rather to prompt students to identify and discuss meaningful mathematical correspondences between strategies.
- This is not a fluency task; the challenges in the task are subtle and should be met using understanding, not by applying recipes.

distributive property; and viewing expressions like 240 × 6 as objects with structure that can be interpreted—not just as instructions to calculate a resulting value.

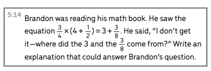
Extending the task

How might students drive the conversation further?

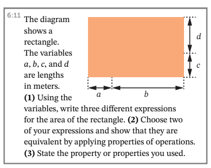
- Students could compare different strategies for solving the problem and identify the mathematical properties that ensure the two strategies lead to the same answer.
- Students could create their own versions of the problem with different numbers. For example, the school's mistake might involve substituting 12-packs for 10-packs...why is the final answer the same in this case as it was for the original problem? In discussing variations, keep the conversation close to the central concepts of the original task.

Related Math Milestones tasks

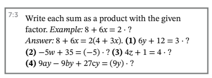
5:14


5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

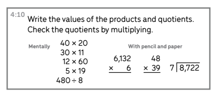
6:11


6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters. **(1)** Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

7:3


7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$
Answer: $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$
(2) $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$
(4) $9ay - 9by + 27cy = (9y) \cdot ?$

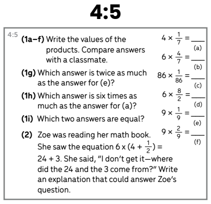
4:10


4:10 Write the values of the products and quotients. Check the quotients by multiplying.

Mentally: 40 × 20, 30 × 11, 12 × 60, 5 × 19, 480 ÷ 8

With pencil and paper: 6,132 × 6; 48 × 39; 7⟌8,722

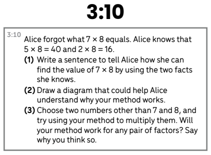
4:5


4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?
(1h) Which answer is six times as much as the answer for (a)?
(1i) Which two answers are equal?
(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

3:10


3:10 Alice forgot what 7 × 8 equals. Alice knows that 5 × 8 = 40 and 2 × 8 = 16.
(1) Write a sentence to tell Alice how she can find the value of 7 × 8 by using the two facts she knows.
(2) Draw a diagram that could help Alice understand why your method works.
(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

Another grade 5 task that prominently features the distributive property is **5:14 Brandon's Equation**.

In later grades, the distributive property will become the central principle in rewriting expressions; see tasks **6:11 Area Expressions** and **7:3 Writing Sums as Products**.

In earlier grades, tasks **4:10 Calculating Products and Quotients, 4:5 Fraction Products and Properties,** and **3:10 Alice's Multiplication Fact** demonstrate how the distributive property is used to do mental math, organize calculations, and extend operations from whole numbers to fractions.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:1? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 5:1? In what specific ways do they differ from 5:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?