5:2 Water Relief

Teacher Notes

Central math concepts

In earlier grades, students might have considered a situation with some resemblance to task 5:2, but with a discrete rather than continuous quantity—such as,

> Neighbors donated 40 picture frames to the 12 residents of a nursing home. That would provide ______ picture frames for each resident and ______ picture frames for the common room.

This situation could be described with an equation model such as

$$40 = 3 \times 12 + 4,$$

where the units of the quotient are "frames per resident" and the units of the remainder are "picture frames." Alternatively, the situation could be described with an equation model such as

$$(40 - 4) \div 12 = 3,$$

where the units of the quotient are again "frames per resident" and the units of the part subtracted are "picture frames." Notice that the only numbers that appear in these two equation models are whole numbers, which is sensible because fractional parts of picture frames aren't part of the situation. Notice also that in the second equation model that uses division, we could check the quotient by multiplying: $3 \times 12 = 40 - 4$ is a true equation.

The equation $40 = 3 \times 12 + 4$ could also describe a continuous quantity like water. In that case, the equation $40 = 3 \times 12 + 4$ says that with a supply of 40 gallons of water, each of 12 residents could be given 3 gallons of water, and there would be 4 gallons left over. However, unlike picture frames, when the quantity in a situation is continuous, we can share the remainder also. That is, the equation

$$40 = 3 \times 12 + 4$$

implies

$$40 \div 12 = (3 \times 12) \div 12 + 4 \div 12,$$

or

$$40 \div 12 = 3 + \tfrac{1}{3}.$$

In this last equation, the units of 3, $\frac{1}{3}$, and $3 + \frac{1}{3}$ are all the same: "gallons per resident." The remainder 4 had units of "gallons," whereas after division by 12, the resulting fractional part $\frac{1}{3}$ has units "gallons per resident."

Task 5:2 thus involves the extension of the number system from whole numbers to fractions. This extension is necessary if we are to share continuous quantities like water equally, and it also solves a conceptual problem that arises as early as grade 3. Consider the dilemma of a third-grade student who solves problems like $72 \div 9 = 8$ but who may wonder what to make of a problem like $72 \div 10 = ?$. The unknown factor can't be

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ___ gallons for each resident.

Answer

$3\frac{1}{3}$ or the equivalent (such as $\frac{10}{3}$).

Click here for a student-facing version of the task.

Refer to the Standards

5.NF.B.3; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

In a challenging life situation like the one described in the task, it would make sense not to waste the $\frac{1}{3}$ fractional part of a gallon per person.

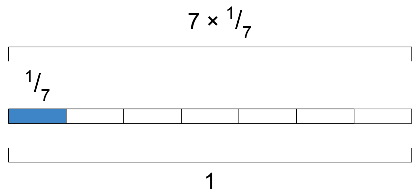
Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:2? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:2? In what specific ways do they differ from 5:2?

7 (because $10 \times 7 = 70$, which is too small); and the unknown factor can't be 8 (because $10 \times 8 = 80$, which is too large). So maybe the unknown factor is between 7 and 8? Closer to 7, because 70 is closer to 72 than 80 is? It isn't until the upper-elementary grades that students fully integrate positive fractions into their expanding system of numbers and operations.

Fractions allow division of any whole number C by any nonzero whole number A. Because $C = A \times \frac{C}{A}$, the quotient $C \div A$ is the number $\frac{C}{A}$, because $C \div A$ is the unknown factor in $C = A \times ?$. It may seem obvious that $\frac{C}{A} = C \div A$ if $\frac{C}{A}$ is already thought of as an instruction to calculate $C \div A$, but the statement $\frac{C}{A} = C \div A$ is more meaningful if the fraction $\frac{C}{A}$ is thought of as a number before it is recognized as a quotient.

An important special case of the principle $\frac{C}{A} = C \div A$ is dividing 1 by a nonzero whole number. For example, $1 \div 7 = \frac{1}{7}$ (see figure). This result can be extended to a quotient like $5 \div 7$ by means of diagrams and/or by thinking that, since 5 is five times as much as 1, then $5 \div 7$ is five times as much as $1 \div 7$, or 5 parts of size $\frac{1}{7}$, which is to say $\frac{5}{7}$.



Extending the number system from the whole numbers to the fractions allows answers to questions like the one our hypothetical third-grade student was wondering about: the answer to $72 \div 10 = ?$ is the number $\frac{72}{10}$.

The relationship between multiplication and division—that $C \div A$ is the unknown factor in $A \times ? = C$—is a trustworthy guide to mathematical reasoning, used frequently beginning in grade 3 and continuing at least until grade 7. The relationship recurs in high school as well, with the study of rational expressions, rational functions, and complex numbers.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying unit fractions; making sense of products of unit fractions; applying ideas of times-as-much in context; and basing multiplicative reasoning on math diagrams.

Extending the task

How might students drive the conversation further?

- Students could check the quotient $3\frac{1}{3}$ or $\frac{10}{3}$ by multiplying.
- Students could work the problem by imagining that the water was donated in bottles that each hold $\frac{1}{3}$ gallon. How many of those bottles were donated? How many bottles does each of the 12 residents get?

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

- Students could describe how the $\frac{1}{3}$ gallon fractional part could be physically provided to the residents (at least approximately), supposing that all the donations were in the form of gallon bottles. How do the quantities in the distribution process correspond to the numbers in the equation 40 = 3 × 12 + 4?

Related Math Milestones tasks

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:5

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10 = ?$	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3} = ?$	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8} = ?$	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5) = ?$	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

Task **5:6 Corner Store** involves fraction products in context. Task **5:13 Frozen Yogurt Machine** is a multi-step word problem that involves multiplicative thinking with fractions. Task **5:5 Calculating** includes the procedural problems 1461 ÷ 6 and 8 × ? = 73 that involve quotients of whole numbers.

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:12

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

In later grades, tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** are word problems that use multiplication and division operations on fractions, completing the extension of arithmetic from whole numbers to fractions. Task **6:12 Is There a Solution? (Multiplication)** looks at this extension from an algebra perspective.

4:5

4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)

$6 \times \frac{4}{7} =$ ___ (b)

$86 \times \frac{1}{86} =$ ___ (c)

$6 \times \frac{8}{2} =$ ___ (d)

$9 \times \frac{1}{9} =$ ___ (e)

$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

3:6

3:6 Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

In earlier grades, task **4:5 Fraction Products and Properties** involves the initial stages of extending multiplication to fractions. The building blocks of fractions are the topic in task **3:6 Unit Fraction Ideas**.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

