5:3 Neighborhood Garden

Teacher Notes

Central math concepts

In grades K–2, students work with length units and concepts of length measurement (CCSS 1.MD.A.2). At first in kindergarten, students make non-numerical comparisons of length and other measurable quantities (CCSS K.MD.A). Then in grade 1, using objects as length units, students learn that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. In grade 2, students extend these ideas to abstract length units and relate length measurement to addition, subtraction, and the number line (CCSS 2.MD.A).

Beginning in grade 3, students learn to recognize area as a measurable attribute of plane figures and to understand concepts of area measurement (CCSS 3.MD.C.5):

- A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
- A plane figure that can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

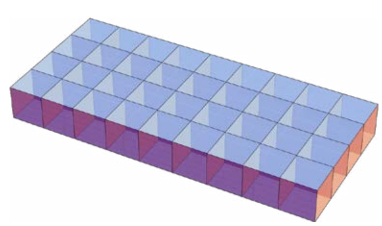
Observe the close parallel to volume in grade 5 (CCSS 5.MD.C.3):

- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- A solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

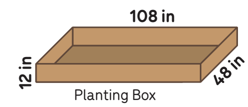
Additional parallels can also be seen for angle measure in grade 4 (CCSS 4.MD.C.5; see the Teacher Notes for task **4:8 Shapes with Given Positions**). Measurement, and especially the idea of a unit, is a coherent and unifying theme in school mathematics.[†]

WIth the length unit as 1 inch, the planting box in task 5:3 could be packed with 12 layers of unit cubes, each layer consisting of an array of unit cubes viewed as 48 rows and 108 unit cubes in each row (see figure; click here for a larger image). Each layer has 48 × 108 unit cubes, and the 12 layers have 48 × 108 × 12 unit cubes total. Thus the volume is 48 × 108 × 12 cubic inches.

Had we chosen the length unit to be 1 foot, then we could pack the planting box with a single array of unit cubes, this array viewed as 4 rows and 9 unit cubes in each row (see figure; click here for a larger image). Thus, the volume is 4 × 9 cubic feet. To emphasize the correspondence with the three-factor volume formula for a rectangular prism,



5:3 A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is 54 ft³ of soil. How many truckloads of soil will fill all of the boxes?



Answer

4 truckloads.

Click here for a student-facing version of the task.

Refer to the Standards

5.MD.A, B; MP.1, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

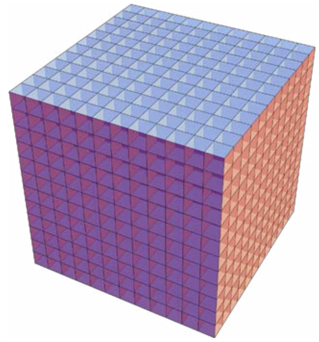
Aspect(s) of rigor:

Application

Additional notes on the design of the task

The numbers in the problem are chosen to have many divisibilities so that, for example, students could solve the problem by analyzing it in the form 6 × 9 × ? = 6 × 4 × 9.

$V = \ell wh$, we could use the multiplicative identity property of 1 to write the product 4 × 9 in the equivalent form 4 × 9 × 1.



Each cubic foot contains 12 × 12 × 12 cubic inches (see figure; click here for a larger image). Seeing each cubic foot as 12 × 12 × 12 cubic inches can help make sense of the fact that 48 × 108 × 12 cubic inches is the same quantity of volume as 4 × 9 × 1 cubic feet:

$$48 \times 108 \times 12 = (12 \times 4) \times (12 \times 9) \times (12 \times 1)$$
$$= (12 \times 12 \times 12) \times (4 \times 9 \times 1).$$

Here the associative and commutative properties of multiplication have been used.

Because the truckloads of soil are given in cubic feet, whereas the planting box dimensions are given in inches, students have to choose which unit to use. Working in units of feet and cubic feet makes the problem much easier to solve, and this is an illustration of how important it is for students to learn they must make choices when solving problems. Students can learn how to choose convenient units in applied problems by experiencing and getting feedback on choosing.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: the equal-groups idea of multiplication; remembering products of single-digit numbers; and finding simple quotients.

Extending the task

How might students drive the conversation further?

- Students might know that, or think about the fact that, soil is often sold by the cubic yard. How many cubic feet is 1 cubic yard? How many cubic yards is one of the truckloads in task 5:3?
- Students might consider making truckload-units central in the problem. How many boxes does a truck hold? ($54 = 36 \times n \Rightarrow n = 54 \div 36 = \frac{54}{36} = \frac{3}{2}$.) Therefore, how many truckloads are needed for 6 boxes? 4 truckloads, because $4 \times \frac{3}{2} = 6$.

Curriculum connection

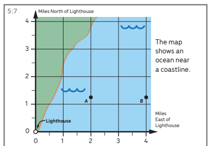
1. In which unit of your curriculum would you expect to find tasks like 5:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:3? In what specific ways do they differ from 5:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See Zimba (2013), "Units, a Unifying Theme in Measurement, Fractions, and Base Ten."

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Related Math Milestones tasks

5:7



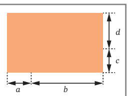
Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle ABCD. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi²?

5:6

(1) Arya and Lily's house is $\frac{1}{2}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{2}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

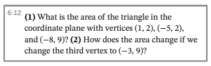
Task **5:7 Shipwrecks** involves geometric measure in a case with fractional dimensions, and task **5:6 Corner Store** involves an equality between two products that depends on the commutative and associative properties of multiplication.

6:11



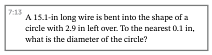
The diagram shows a rectangle. The variables *a*, *b*, *c*, and *d* are lengths in meters. **(1)** Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

6:12



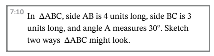
(1) What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

7:13



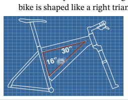
A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

7:10



In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

8:3



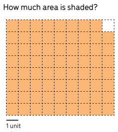
On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.

8:12

Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m³ of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

In later grades, task **6:11 Area Expressions** involves a geometric measure in a case where the lengths are variables rather than numbers; task **6:12 Coordinate Triangle** involves area measure for a triangle; task **7:13 Wire Circle** involves a geometric measure of a curvilinear object; task **7:10 Triangle Conditions** involves length and angle measures in a triangle; task **8:3 Bicycle Blueprint** involves the Pythagorean theorem, and task **8:12 Fish Tank Design** involves volume measurement for a quarter-cylinder.

3:3



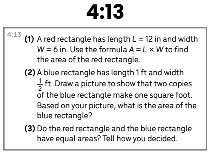
(1) How much area is shaded?

1 unit of length

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ________ Width: ________

4:13



(1) A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

(2) A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

In earlier grades, tasks **3:3 Length and Area Quantities** and **4:13 Area Units** involve concepts of area measurement.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

