5:5 Calculating

Teacher Notes

Math Milestones™

Central math concepts

Task 5:5 focuses on procedures. For each individual computation included in task 5:5, students have a choice between using an algorithm or a strategy.

Algorithms are usefully distinguished from strategies (CCSS, Glossary; see figure). Strategies are "purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another." Mental calculation often uses strategies. For example, we could calculate 0.75 ÷ 0.01 mentally by using understanding of place value to replace the given quotient by the equivalent 75 ÷ 1. Or we could calculate $3 \div \frac{1}{8}$ by thinking that since there are 8 eighths in every 1, there must be 24 eighths in 3. Flexible procedural fluency based on place value, properties of operations, relationships between operations, and ideas of equivalence is valuable both in practical terms and also as a way of doing arithmetic that prefigures algebra. In algebra, strategies are the most common way of working.

> **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
>
> **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Algorithms are inflexible by definition. One step follows another in the prescribed order. Some algorithms are also much simpler to execute than others. The standard multi-digit addition algorithm is less complex than the standard multi-digit subtraction algorithm. The standard algorithm for dividing fractions is less complex than the standard algorithm for dividing multi-digit numbers.

Sometimes even an efficient general-purpose algorithm wouldn't be an efficient approach to a particular instance of a calculation, as in a subtraction problem like 4,003 - 8. Instead of applying the algorithm $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$ to the problem $\frac{1}{2} + \frac{1}{3} - \frac{1}{5}$ in task 5:5, one can use the principle of fraction equivalence to replace the given problem with a problem in which the denominators are equal: $\frac{15}{30} + \frac{10}{30} - \frac{6}{30}$.

On the other hand, when faced with a calculation, there may be times when we don't find ourselves readily inventing a flexible mental procedure on the spot, so it's valuable to know and be proficient with an algorithm.

An important value in mathematics education is that of being able to solve problems in multiple ways. This brings the pleasures of seeing how a coherent subject holds together, and it allows students to check answers, unify their understanding of concepts, and learn from different ways of thinking that emerge in the classroom community. A parallel but also important outcome of mathematics education is for students to be supported in gaining procedural fluency with algorithms for the actually

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10 = ?$	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3} = ?$	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8} = ?$	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5) = ?$	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

Answer

4087 × 53 = **216,611**
246 × 914 = **224,844**
9744 ÷ 12 = **812**
1461 ÷ 6 = $\mathbf{243\frac{1}{2}}$ **or 243.5**
4 - (8 - 4) = **0**

$\frac{1}{10} \div 10 = \mathbf{\frac{1}{100}}$
$\frac{7}{8} \times \frac{5}{3} = \mathbf{\frac{35}{24}}$
8 × ? = 73: ? = $\mathbf{\frac{73}{8}}$ or $\mathbf{9\frac{1}{8}}$
$3 \div \frac{1}{8} = \mathbf{24}$
$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \mathbf{\frac{19}{30}}$
$\frac{1}{3} \div (6 \times 5) = \mathbf{\frac{1}{90}}$

0.4 × 0.9 = **0.36**
0.75 ÷ 0.01 = **75**
0.63 ÷ 0.3 = **2.1**
0.86 + 0.4 = **1.26**
0.72 - 0.17 = **0.55**
0.02 + 0.2 = **0.22**
0.8 - 0.55 = **0.25**
637 - 1.31 = **635.69**

Click here for a student-facing version of the task.

Refer to the Standards

5.NBT.B, 5.NF.A, B; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

quite small set of recurrent problem types for which an algorithm exists. This small set can be found in the CCSS-M by searching for “algorithm.”

Many people are not perfectionists in execution; this doesn’t mean they should be identified as “bad at math.” Anxiety and low confidence can also contribute to errors. Thus, errors in calculations are sometimes the result of careless mistakes rather than gaps in understanding. To reduce the rate of errors, students can learn general and specific debugging skills. Debugging involves reflecting on the execution of the specific mistake and formulating a behavior that is a correct execution for similar situations. This can be much more efficient than reteaching topics.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value, number sense of decimals, and decimal notation; number sense of fractions; multiplication facts and related quotients; and the relationship between multiplication and division.

Extending the task

How might students drive the conversation further?

- Checking differences by adding, and checking quotients by multiplying, can offer additional procedural practice, reinforce the relationship between addition and subtraction ($C - A$ is the unknown factor in $A + \square = C$), and reinforce the relationship between multiplication and division ($C \div A$ is the unknown factor in $A \times \square = C$).
- Students could make sense of their answers another way by making estimates of the values; for example, 246 × 914 should be somewhat less than 246 × 1,000 = 246,000; 0.63 ÷ 0.3 should be slightly greater than 0.6 ÷ 0.3 = 2.

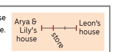
Related Math Milestones tasks

5:4

5:4 **(1)** Circle T for true or F for false.
(a) 9 thousandths + 5 hundredths > 3 hundredths + 2 tenths T F
(b) 92 hundredths + 4 thousandths > 0.924 T F
(c) 0.456 < 0.5 T F
(2) Write each number in the requested form.
(a) 7 thousandths + 5 tenths = _____ (decimal)
(b) 0.1 tenths = _____ (decimal)
(c) $\frac{2}{100} + \frac{5}{1000}$ = _____ (decimal) = _____ (fraction in lowest terms)

5:6

5:6 **(1)** Arya and Lily’s house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon’s house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Task **5:4 Place Value to Thousandths** involves decimal place value concepts, while tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** involve fraction multiplication and division in context.

6:14

6:14 *Pencil and paper* **(1)** 81.53 ÷ 3.1 = ? **(2)** $\frac{7}{8} \div \frac{2}{3}$ = ? **(3)** Check both of your answers by multiplying.

In later grades, task **6:14 Dividing Decimals and Fractions** marks the culmination of algorithmic procedures with general fraction and decimal division.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

- Some of the problems where buggy execution may occur could include 4087 × 53 (because of the 0 digit), 4 - (8 - 4) (because of the subtraction sign in front of parentheses), and the decimal problems in which the two operands have differing numbers of digits.
- The task does not require students to show their work. For some of the calculations, students might simply write down the answer by inspection. For other calculations, students might write out steps along the way to finding the result.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:5? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 5:5? In what specific ways do they differ from 5:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

4:10

4:10 Write the values of the products and quotients.
Check the quotients by multiplying.

Mentally

40×20

30×11

12×60

5×19

$480 \div 8$

With pencil and paper

$6{,}132 \times 6$ $\quad$ 48×39 $\quad$ $7\overline{)8{,}722}$

4:14

4:14 $540{,}909 + 87{,}808 - 5{,}864 + 2{,}556 = ?$

In earlier grades, task **4:10 Calculating Products and Quotients** includes a range of procedural tasks amenable to grade-level techniques. Task **4:14 Fluency with Multi-Digit Sums and Differences** involves the culmination of multi-digit addition and subtraction procedures.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

