5:6 Corner Store

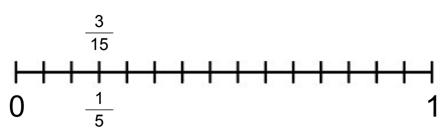
Teacher Notes

Central math concepts

Part (1)(a) of task 5:6 involves multiplying the unit fractions $\frac{1}{3}$ and $\frac{1}{5}$ in context. Making sense of a product of unit fractions is a special moment in a student's mathematical education, a moment of entry into a newly sophisticated world of interrelated numbers and operations. A product like $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$ doesn't sit comfortably alongside third-grade understandings of products—such as, say, interpreting 6 × 5 as the total number of marbles in 6 bags if each bag contains 5 marbles. For one thing, the product $\frac{1}{3} \times \frac{1}{5}$ is less than either of its factors, something that never happens with marbles. And yet multiplication is a single operation with a meaning that transcends the size or format of the numbers being multiplied. Synthesizing students' intuitions about multiplication is part of the important work in grade 5.

Fortunately, conceptual bridges are available to help students make the journey from multiplication ideas that only work for whole numbers to multiplication ideas that work for all numbers. One such bridge is using representations like area models and number lines, which are well suited to both whole-number products and products involving fractions. Fraction equivalence is a powerful tool for reasoning about fraction operations. Another bridge is the multiplicative concept of "times as much," which could be seen as intermediate between the grade 3 discrete equal-groups concept of multiplication and the grade 5 scaling concept of multiplication. Times-as-much thinking is well suited to contexts involving continuous measurement quantities like length, time, and mass or weight, and quantities derived from these by multiplication and division. These are the kinds of quantities for which fractional parts most naturally arise.

The diagram shows an equivalent-fractions approach to making sense of the product $\frac{1}{3} \times \frac{1}{5}$, using a length model to organize the thinking.

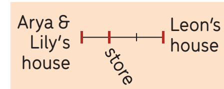


One-third of $^1/_5$ means 1 part of a partition of $^1/_5$ into 3 equal parts. To facilitate partitioning $^1/_5$ into 3 equal parts, one can use the fraction equivalence principle to rewrite $^1/_5$ as $^3/_{15}$. Because $^1/_5$ is now expressed as 3 of something (namely 3 fifteenths), it is more apparent that in a partition of $^1/_5$ into 3 parts, 1 of the parts will have size $^1/_{15}$. Thus, $^1/_3 \times {}^1/_5 = {}^1/_{15}$.

Eventually the kinds of reasoning processes reflected in the diagram cease to be the focus of a student's work with fractions, and calculating products of fractions becomes a routine exercise in procedural fluency. However, extending multiplication and division from whole numbers to fractions is a momentous mathematical journey that begins not with procedure but with reasoning, sense-making, and quantitative thinking.

Part (2) of task 5:6 involves the equality of the products $\frac{2}{3} \times \frac{1}{5}$ and $\frac{1}{3} \times \frac{2}{5}$. If we think of the first product as $2 \times (\frac{1}{3} \times \frac{1}{5})$ and the second product as

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



Answer

(1) (a) $\frac{1}{15}$ mi. **(b)** $\frac{2}{15}$ mi.

(2) (a) $\frac{2}{15}$ mi. **(b)** Leon and Lily ran the same distance measured in miles. Explanations for this will vary. One kind of explanation could involve recognizing that doubling and then halving a given length results in the given length. Another kind of explanation could involve the algorithm for multiplying fractions. Answers may include such explanatory techniques as showing a math diagram (for example, an area model, tape diagram, or number line), writing expressions and equations, or using analogies (for example, "If I my water bottle is $\frac{1}{3}$ full and yours is $\frac{2}{3}$ full, but my water bottle is twice as big as yours, then we have the same amount of water").

Click here for a student-facing version of the task.

Refer to the Standards

5.NF.B.4a, 5.NF.B.6; MP.2, MP.3, MP.4, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

$\frac{1}{3} \times (2 \times \frac{1}{5})$, then the equality $\frac{1}{3} \times (2 \times \frac{1}{5}) = 2 \times (\frac{1}{3} \times \frac{1}{5})$ reflects the fact that one-third of twice a given quantity is twice as much as one-third of the quantity. (There would be no preference between two-thirds of one sum of money and one-third of another sum of money, provided the second sum of money were twice as large.) This could again be justified by a diagram and/or by using equivalent fractions, and viewed as an example of the associative and commutative properties of multiplication:

$$\frac{2}{3} \times \frac{1}{5} = (2 \times \frac{1}{3}) \times \frac{1}{5} = (\frac{1}{3} \times 2) \times \frac{1}{5} = \frac{1}{3} \times (2 \times \frac{1}{5}) = \frac{1}{3} \times \frac{2}{5}.$$

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying unit fractions; making sense of products of unit fractions; applying ideas of times-as-much in context; and basing multiplicative reasoning on math diagrams.

Extending the task

How might students drive the conversation further?

- Students who gave different accounts in part (2)(b), or used different representations, could look for correspondences between them.
- Students could ask and answer additional questions about the situation, such as, "If Leon walked halfway from his house to Arya's house, how far from the store would Leon be?"

Related Math Milestones tasks

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:2

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

5:11

5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
(1) Is Juliet thinking of something possible or impossible? Use math to decide for sure.
(2) Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

Task **5:13 Frozen Yogurt Machine** is a multi-step word problem that involves multiplicative thinking with fractions. Task **5:2 Water Relief** involves interpreting the quotient of two whole numbers as a fraction. Task **5:11 Juliet's Rectangle** could promote thinking about the extension of multiplication from whole numbers to fractions.

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

In later grades, tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** are word problems that use multiplication and division operations on fractions, completing the extension of arithmetic from whole numbers to fractions. Task **6:6 Planting Corn** is about proportionality (can involve application of times-as-much thinking). In task **6:13 Is There a Solution? (Multiplication)**, the equation can be made sense of as a question about multiplicative scaling.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

In part (2)(b), the phrasing "why is it true?" is intended to invite sense-making about the observation that Leon and Lily ran the same distance measured in miles. The intent is to push beyond an algorithmic demonstration that $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15} = \frac{1}{3} \times \frac{2}{5}$ and think about the sizes of the quantities and the meaning of the operation of multiplication.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:6? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:6? In what specific ways do they differ from 5:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

4:5

4:5

(1a–f) Write the values of the products. Compare answers with a classmate.

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

$4 \times \frac{1}{7} =$ ____ (a)

$6 \times \frac{4}{7} =$ ____ (b)

$86 \times \frac{1}{86} =$ ____ (c)

$6 \times \frac{8}{2} =$ ____ (d)

$9 \times \frac{1}{9} =$ ____ (e)

$9 \times \frac{2}{9} =$ ____ (f)

4:1

4:1

A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

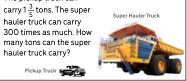
Equation model: ____________

Answer: ____________

4:12

4:12

The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?



3:6

3:6

Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

In earlier grades, task **4:5 Fraction Products and Properties** can involve times-as-much thinking and sense-making about fraction products. Tasks **4:1 A Tablespoon of Oil** (whole numbers) and **4:12 Super Hauler Truck** (fractions) are word problems involving multiplicative comparison. The building blocks of fractions are the topic in task **3:6 Unit Fraction Ideas**.

5:6 Corner Store

Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?