# 5:7 Shipwrecks

Teacher Notes

## Central math concepts

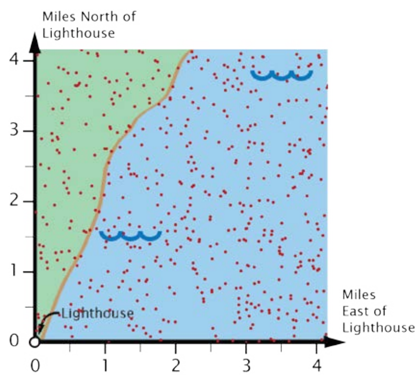
The coordinate plane is a relatively recent invention in the history of mathematics. Introduced in the 17th Century, this representation scheme was revolutionary for the way it connected algebra and geometry. Today the coordinate plane is an essential representation scheme for functions, two-variable equations, and bivariate data sets. Moreover, any time a computer is used for geometric or spatial applications, such as for GPS, robot vision, or finding shortest routes, those problems are being represented within the computer by means of numerical coordinates for the locations and boundary points of shapes. (A computer doesn't "see" a rectangle, but it can store and manipulate the rectangle in the form of a list of four ordered pairs of numbers.)

In task 5:7, the $(x, y)$ coordinates of a point have a spatial representation as map coordinates. In later grades, where contexts will often involve a proportional relationship between two covarying quantities, the $(x, y)$ coordinates of a point will often be a particular pair of values taken on by the quantities as they covary.

**Connection to the number line**. In a number line, every point in the continuum of the line is imagined to be labeled with a real number. In a coordinate plane, every point in the continuum of the plane is imagined to be labeled with an ordered pair of real numbers.

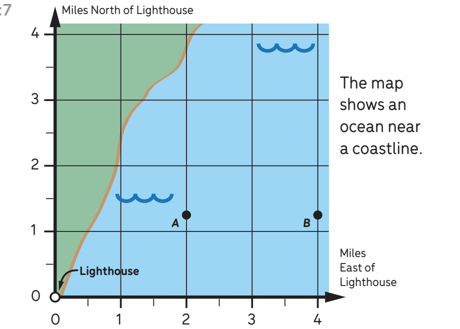
In a coordinate plane, the two coordinate axes are number lines. In contextual problems, each coordinate axis (number line) can be thought of as a measurement scale with a unit. In task 5:7, the unit for each number line is 1 mile, but in general the units on the two axes don't have to have the same value or even be quantities of the same type. For example, in a distance-time graph, the unit on the horizontal axis might be 1 hour while the unit on the vertical axis might be 1 mile.

Gridlines on a coordinate plane are the two-dimensional extension of the tick marks on a number line. Gridlines can make it easier to determine the coordinates of points by visual inspection. But it must be understood that a coordinate plane includes all of the plane's infinitely many points, not just the special points found at the intersections of gridlines (see figure). Likewise, a number line contains all of the line's infinitely many points, not just the points indicated by tick marks.




Dots show the locations of 500 randomly chosen points in the part of the coordinate plane used in task 5:7.

5:7

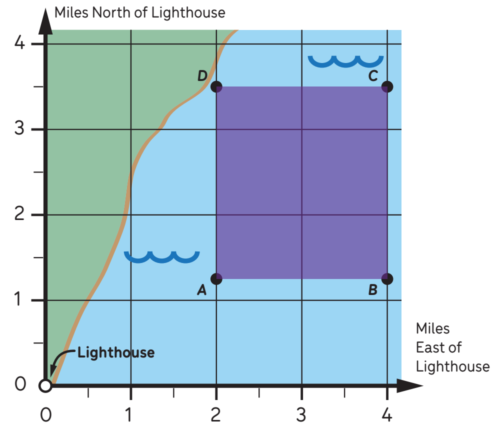



The map shows an ocean near a coastline.

Shipwrecks are at locations $A$ $(2, 1\frac{1}{4})$ and $B$ $(4, 1\frac{1}{4})$. Shipwrecks are also at locations $C$ $(4, 3\frac{1}{2})$ and $D$ $(2, 3\frac{1}{2})$. **(1)** Mark $C$ and $D$ on the map and shade rectangle *ABCD*. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in $\text{mi}^2$?

### Answer

**(1)** See figure. **(2)** $4\frac{1}{2}$ $\text{mi}^2$.




[Click here](#) for a student-facing version of the task.

### Refer to the Standards

5.NF.B.4b, 5.G.A; MP.2, MP.4, MP.7. Standards codes refer to [www.corestandards.org](http://www.corestandards.org). One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

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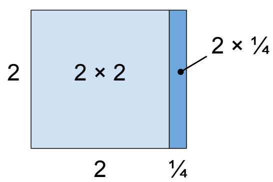
## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding the meaning of 1 square unit of area; relating number lines to quantities in context; and doubling a mixed number or fraction.

## Extending the task

How might students drive the conversation further?

- If students determined the area of the shaded rectangle using multiple strategies or thought processes, students could connect those approaches to each other and to the rectangle area formula. According to the formula, the area in square miles equals the product $2 \times 2\frac{1}{4}$, or equivalently the product $2 \times \frac{9}{4}$.
    - Using distributive property thinking with the mixed number factor, the product $2 \times (2 + \frac{1}{4})$ equals the sum $2 \times 2 + 2 \times \frac{1}{4} = 4 + \frac{1}{2}$. The terms in this equation can be identified in an area diagram (see figure).

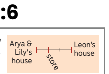



    - Alternatively, using unit thinking with the product $2 \times \frac{9}{4}$, one could make sense of the product $2 \times \frac{9}{4} = \frac{9}{2}$ by thinking, "I could double 9 fourths by replacing the unit of fourths with a unit twice as large, making 9 halves."
    - Of course, the fraction product algorithm $2 \times \frac{9}{4} = \frac{2}{1} \times \frac{9}{4} = \frac{18}{4}$ gives the result too.
- Students could draw additional tick marks on the coordinate axes to represent fourths of a mile, then add corresponding gridlines. What do students notice, wonder, or want to calculate about this picture? For example, how many of the resulting grid squares are in the shaded rectangle? What is the area of each of those grid squares in square miles? How do those numbers determine the area of the shaded rectangle?

## Related Math Milestones tasks

**5:6**

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?




**5:11**

5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
**(1)** Is Juliet thinking of something possible or impossible? Use math to decide for sure.
**(2)** Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

**5:14**

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

**5:5**

5:5 Write the requested values.

| | | |
|---|---|---|
| $4087 \times 53 = ?$ | $\frac{1}{10} \div 10 = ?$ | $0.4 \times 0.9 = ?$ |
| $246 \times 914 = ?$ | $\frac{7}{8} \times \frac{5}{3} = ?$ | $0.75 \div 0.01 = ?$ |
| $9744 \div 12 = ?$ | $8 \times ? = 73$ | $0.63 \div 0.3 = ?$ |
| $1461 \div 6 = ?$ | $3 \div \frac{1}{8} = ?$ | $0.86 + 0.4 = ?$ |
| $4 - (8 - 4) = ?$ | $\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$ | $0.72 - 0.17 = ?$ |
| | $\frac{1}{3} \div (6 \times 5) = ?$ | $0.02 + 0.2 = ?$ |
| | | $0.8 - 0.55 = ?$ |
| | | $637 - 1.31 = ?$ |

Fraction products receive a conceptual emphasis in tasks **5:6 Corner Store**, **5:11 Juliet's Rectangle**, and **5:14 Brandon's Equation**, and a procedural emphasis in task **5:5 Calculating**.

### Aspect(s) of rigor:

Concepts, Application

### Additional notes on the design of the task

- The coordinate axes are shown as heavy lines in order to emphasize the connection to number lines.
- On the spatial scale of the problem (4 miles × 4 miles), objects the size of a shipwreck or a lighthouse are well modeled as single points.

### Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:7? In what specific ways do they differ from 5:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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**6:3**

6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

| Time | Hours after Midnight | Temp °F |
|---|---|---|
| 8:00 pm | –4 | –42 |
| 9:00 pm | –3 | –42 |
| 10:00 pm | –2 | –41 |
| 11:00 pm | –1 | –40 |
| Midnight | 0 | –39 |
| 1:00 am | 1 | –39 |
| 2:00 am | 2 | –38 |

Plot the data on graph paper and label the plot. Describe any patterns you see.

**6:4**

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

**6:12**

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

In later grades, task **6:3 South Pole Temperatures** involves using the coordinate plane to display a set of bivariate measurement data. Task **6:4 Gas Mileage** (part (4)) involves using the coordinate plane to plot values from a proportional relationship. Task **6:12 Coordinate Triangle** places a triangle in the coordinate plane.

**4:13**

4:13 **(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.
**(2)** A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?
**(3)** Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

**4:5**

4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.
**(1g)** Which answer is twice as much as the answer for (e)?
**(1h)** Which answer is six times as much as the answer for (a)?
**(1i)** Which two answers are equal?
**(2)** Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

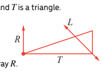
$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)

**4:12**

4:12 The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?


**4:8**

4:8 $L$ is a line, $R$ is a ray, and $T$ is a triangle. True or false:
**(1)** Line $L$ is a line of symmetry for triangle $T$.
**(2)** Line $L$ intersects ray $R$.
**(3)** Triangle $T$ has two angles measuring less than 90 degrees.




In earlier grades, task **4:13 Area Units** involves rectangle area in a case of a fractional length. Task **4:5 Fraction Products and Properties** involves multiplying a fraction by a whole number with a conceptual emphasis, and task **4:12 Super Hauler Truck** involves a multiplicative comparison leading to multiplying a fraction by a whole number in context. Task **4:8 Shapes with Given Positions** involves the distinction between a geometric (mathematical) object and the physical diagram that refers to it.

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# 5:7 Shipwrecks

Teacher Notes

## Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

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