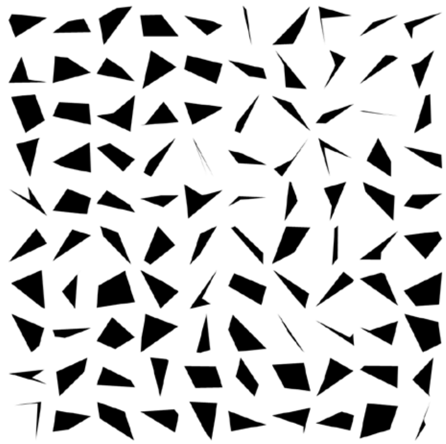
5:8 Alana's New Shape Category

Teacher Notes

Central math concepts

Mathematics is a creative endeavor. Definitions can be invented, and then the consequences of those definitions can be discovered through mathematical reasoning. Anyone has the power to make mathematics!



The study of geometry tends to concentrate on figures with special properties, such as isosceles triangles, parallelograms, or regular polygons. These figures aren't representative of the larger classes to which they belong; for example, three points placed at random in the plane almost never define an isosceles triangle. Special figures are worth studying for many reasons, including the way their special features allow us to draw further conclusions about them. However, it is also worth acknowledging that these shapes are atypical. For example, the figure shows 100 quadrilaterals randomly generated by a computer. None of the quadrilaterals are squares, rhombuses, rectangles, parallelograms, or trapezoids.

All of them, however, are alana-gons!

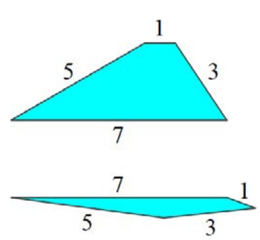
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: remembering definitions of special quadrilaterals; reasoning about figures based on attributes; drawing examples of special quadrilaterals; and basing mathematical arguments on figures.

Extending the task

How might students drive the conversation further?

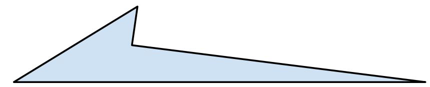
- Students could toss four crumpled-up balls of paper onto the floor to mark four locations, then connect the locations with string, cash register tape, or similar. Then check: is the resulting figure an alana-gon?
- Students could investigate whether there is a hexagon with all six sides of different lengths.
- Given the two alana-gons shown (which have equal perimeters of 16 units), students could decide by inspection which alana-gon has the greater area (it is the one on top in the figure).



5:8 A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths. **(1)** Draw an example of an alana-gon. **(2)** True or false: **(a)** All squares are alana-gons. **(b)** No trapezoids are alana-gons.

Answer

(1) Answers may vary; see example.
(2) **(a)** False. **(b)** False.



Click here for a student-facing version of the task.

Refer to the Standards

5.G.B; MP.3, MP.6, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

It is intentional that the task tells the story of a student, Alana, rather than asking in the abstract about whether squares or trapezoids belong to an identified category. This is intended to emphasize the possibilities for creativity and invention in making mathematics.

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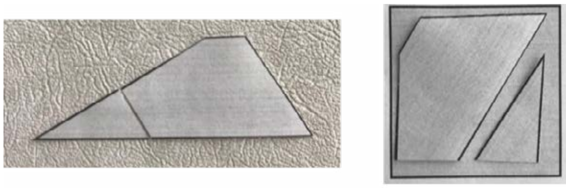
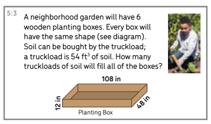
- Students could try to draw a quadrilateral with the same perimeter as the top alana-gon, but with greater area. (The image shows one answer: a square with side length 4 units has greater area than the top alana-gon.)



After cutting the alana-gon into two pieces as shown, both pieces fit in a square of side length 4 units, with area left over.

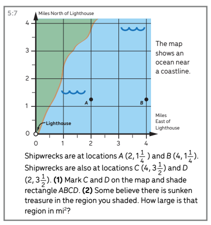
Related Math Milestones tasks

5:3



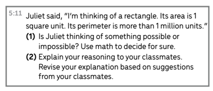
5:3 A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is 54 ft^3 of soil. How many truckloads of soil will fill all of the boxes?

5:7



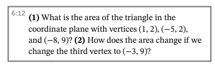
Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:11



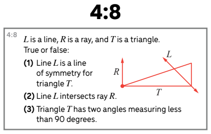
5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
(1) Is Juliet thinking of something possible or impossible? Use math to decide for sure.
(2) Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

6:12



6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices $(1, 2)$, $(-5, 2)$, and $(-8, 9)$? **(2)** How does the area change if we change the third vertex to $(-3, 9)$?

4:8



4:8 L is a line, R is a ray, and T is a triangle. True or false:
(1) Line L is a line of symmetry for triangle T.
(2) Line L intersects ray R.
(3) Triangle T has two angles measuring less than 90 degrees.

Tasks **5:3 Neighborhood Garden**, **5:7 Shipwrecks**, and **5:11 Juliet's Rectangle** involve geometric measurement.

In later grades, task **6:12 Coordinate Triangle** places a geometric figure in the coordinate plane.

In earlier grades, task **4:8 Shapes with Given Positions** involves definitions of geometric figures and their measures.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:8? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:8? In what specific ways do they differ from 5:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

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