# 5:9 Walkathon

Teacher Notes

## Central math concepts

Based on interviews with middle-grades students,[†] education researcher Larry Sowder listed seven common strategies used by students to solve word problems. Ordered roughly from the least desirable to the most desirable, the strategies Sowder observed were as follows:

1. Find the numbers and add (or multiply or subtract...; the choice may be dictated by what has taken place in class recently or by what operation the student feels most competent at doing).
2. Guess at the operation to be used.
3. Look at the numbers; they will "tell" you which operation to use (e.g., "...if it's like, 78 and maybe 54, then I'd probably either add or multiply. But [78 and] 3, it looks like a division problem because of the size of the numbers").
4. Try all the operations and choose the most reasonable answer.
5. Look for isolated "key" words or phrases to tell which operations to use (e.g., "all together" means to add).
6. Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication and choose the more reasonable answer. If smaller, try both subtraction and division and choose the more reasonable answer.
7. Choose the operation whose meaning fits the story.

Especially when a word problem involves fractional quantities, the only robust strategy on Sowder's list is the last strategy: choosing the operation whose meaning fits the story. Single-step problems are useful for learning the core meanings and uses of the operations so that students can grow mathematically into using all four operations flexibly when solving multi-step problems. For multi-step problems, success in problem solving depends on understanding the meanings and uses of addition, subtraction, multiplication, and division, and the relationships between the operations. Bear in mind also that beyond grade 5, solving problems in middle grades will involve using algebra to calculate with variables as if they were numbers. For such problems, students will not be able to rely on looking at the numbers to "tell" them what operation to use. Understanding the meanings of the operations is therefore valuable as preparation not only for powerful problem solving in the elementary grades, but also as preparation for algebra.

In particular, task 5:9 involves addition and subtraction and the relationship between them. The mathematical relationship between addition and subtraction is that $C - A$ is the unknown addend in $A + ? = C$. (One might paraphrase this statement by saying that, "Given a total and one part, subtraction finds the other part.") Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

5:9 On Saturday there was a walkathon.

Catherine

I walked $\frac{1}{3}$ mile farther than Leslie.

I walked $1\frac{1}{4}$ mile.

How many miles did Leslie walk?

### Answer

$\frac{11}{12}$ mi.

Click here for a student-facing version of the task.

### Refer to the Standards

5.NF.A.1, 2; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

### Aspect(s) of rigor:

Application, Procedural skill and fluency

### Additional notes on the design of the task

Word problems involving Compare situations can sometimes consist of complex text. Therefore, task 5:9 presents the given information in the form of a monologue by Catherine. This could also invite an approach of having students convey the task to each other by reciting the monologue.

### Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 5:9? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 5:9? In what specific ways do they differ from 5:9?

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between addition and subtraction can play out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:[‡]

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

In particular, the situation type in task 5:9 is called "Compare with Smaller Unknown." It is a Compare situation because Catherine is using subtraction to compare her distance with Leslie's distance; and more specifically, the situation is "Compare with Smaller Unknown" because the initially unknown quantity is how far Leslie walked (and Leslie walked the shorter distance). During the primary grades, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are extended into working with fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables.

The difference $1\frac{1}{4} - \frac{1}{3}$ could be calculated in many ways. All methods will involve using equivalent fractions to replace the given problem with one in which the denominators are equal (see figure). Transforming a problem to make it easier to solve is one of the most powerful mathematical practices of all.

$$\begin{array}{ccc} 1\frac{1}{4} & - & \frac{1}{3} \\ \shortparallel & & \shortparallel \\ \frac{15}{12} & - & \frac{4}{12} \end{array}$$

Adding and subtracting with unequal denominators need not be approached as an algorithmic process. For example, a student could rewrite the problem $1\frac{1}{4} - \frac{1}{3}$ as $\frac{5}{4} - \frac{1}{3}$ and then use fraction equivalence to replace $\frac{5}{4}$ with its equal $\frac{15}{12}$ and replace $\frac{1}{3}$ with its equal $\frac{4}{12}$; this gives $\frac{5}{4} - \frac{1}{3} = \frac{15}{12} - \frac{4}{12}$. Alternatively, a student could compare $\frac{1}{3}$ and $\frac{1}{4}$ in the equivalent forms $\frac{4}{12}$ and $\frac{3}{12}$, recognizing that $\frac{1}{3}$ is greater than $\frac{1}{4}$ by an amount $\frac{1}{12}$, and therefore the answer to the task is $1 - \frac{1}{12}$. Replacing the problem $1\frac{1}{4} - \frac{1}{3}$ with the equivalent problem $1 - \frac{1}{12}$ is analogous to a mental calculation of a difference in a case like 1,002 - 7, in which case we could think of the equivalent problem 1,000 - 5.

Word problems vary considerably in the uses to which they put the basic operations, and they also vary in the complexity of the calculation

### Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 5:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Sowder, Larry. (1988). *Concept-Driven Strategies for Solving Problems in Mathematics. Final Project Report*. San Diego State Univ., CA. Center for Research in Mathematics and Science Education. https://files.eric.ed.gov/fulltext/ED290629.pdf

‡ See Table 2, p. 9 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. Students for whom the calculation $1\frac{1}{4} - \frac{1}{3}$ is time-consuming and/or effortful may need to be redirected to the context after obtaining the result $1\frac{1}{4} - \frac{1}{3} = \frac{11}{12}$, so as to relate the numbers in this equation to the context and answer the question in the task.

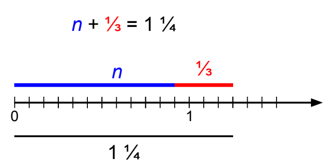
## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: generating equivalent fractions; subtracting fractions with equal denominators; working with mixed numbers; using number sense of fraction size; and representing addition and subtraction on a number line.

## Extending the task

How might students drive the conversation further?

- Students could be asked to write an equation model for the situation, such as $n + \frac{1}{3} = 1\frac{1}{4}$. The equation could be represented on a number line, as shown in the figure.




- Students could check the reasonableness of their answers, or predict the approximate value of the answer, by estimating that the answer will be close to 1 mi since $\frac{1}{3}$ and $\frac{1}{4}$ are close in value.
- Students could be asked to imagine that the path for the walkathon had been marked with flags every twelfth of a mile. In that case, how many flags did Catherine pass? How many flags did Leslie pass? How many more flags did Catherine pass than Leslie? These answers could be used to create a new version of the problem (keeping all the distances the same): "I passed ______ more flags than Catherine. I passed ______ flags. How many flags did Leslie pass?"

## Related Math Milestones tasks

**5:10**

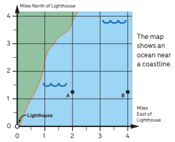
5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$

**(2)** Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.

**(3)** Show one of the above problems and its solution on a number line.

**5:7**

5:7



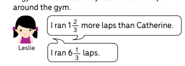

Shipwrecks are at locations *A* $(2, 1\frac{1}{4})$ and *B* $(4, 1\frac{1}{4})$. Shipwrecks are also at locations *C* $(4, 3\frac{1}{2})$ and *D* $(2, 3\frac{1}{2})$. **(1)** Mark *C* and *D* on the map and shade rectangle *ABCD*. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in $mi^2$?

**7:5**

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

**4:9**

4:9 In gym it was fitness day. Students ran laps around the gym.




How many laps did Catherine run?

Task **5:10 Number System, Number Line** (part (1)) features an unknown addend problem involving both a fraction and a decimal. Task **5:7 Shipwrecks** involves finding the difference between $3\frac{1}{2}$ and $1\frac{1}{4}$.

In later grades, task **7:5 Is There a Solution? (Addition)** involves an equation for an unknown addend in the rational numbers.

In earlier grades, task **4:9 Fitness Day** is a word problem of situation type “Compare with Smaller Quantity Unknown” in which the calculation is similar to the one in task 5:9, except with equal denominators.

# 5:9 Walkathon

Teacher Notes

## Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?