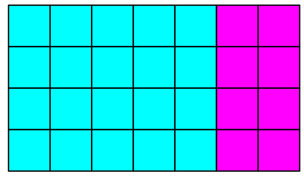
6:10 Weekdays and Weekend Days

Teacher Notes

Central math concepts

If we say that "The ratio of weekdays to weekend days in February 2021, was 20:8," then we haven't said much more than that there were 20 weekdays and 8 weekend days that month. The word "ratio" isn't doing much work in that sentence. On the other hand, if we say that the ratio of weekdays to weekend days in February 2021, was 5:2, then we are invoking ideas of scaling: it's not that there were 5 weekdays and 2 weekend days that month, but rather that *for every* 5 weekdays there were 2 weekend days.



Schematic of days that month

The phrase "for every" in the previous sentence is reminiscent of the common multiplication phrase "for each"—as in, "For each tutor there were 2 students." Just as the total number of students is double the number of tutors, the total number of weekend days is double the number of groups of 5 weekdays in the month. Equivalently, the total number of weekend days is $\frac{2}{5}$ of the number of weekdays. Reversing that comparison, the total number of weekdays is $\frac{5}{2}$ of the number of weekend days.

In a purely technical sense, the ratio a:b isn't a number, because it's a symbol that consists of two numbers separated by a colon. But always to insist on that level of precision in language would be counterproductive, seeing as it's very common in mathematics to say such things as, "The ratio of a circle's circumference to its diameter is π" (and π is a number), or "The slope of a line in the coordinate plane is the ratio of the rise over the run" (and slope is a number). Such usages are reasonable and productive in view of the fact that ratio a:b can certainly be said to have a value, namely the number $\frac{a}{b}$ (or equivalently, the numerical result of the division $a \div b$). Two ratios that are equivalent to a:b, say $(4a)$:$(4b)$ and $(2a)$:$(2b)$, have equal values, because $\frac{4a}{4b}$ and $\frac{2a}{2b}$ are equal as numbers.

In a table of equivalent ratios, all the ratios have the same value. That connects constant ratios to their important sequel, proportional relationships. As an example in context, in a recipe for dry rub that specifies 4 tablespoons of paprika and 2 tablespoons of brown sugar, the ratio of paprika to brown sugar is constant, which guarantees a constant paprika-brown-sugar flavor profile no matter how large or small the quantity of dry rub we make. If we graphed the amount of paprika against the amount of brown sugar for different total quantities of dry rub, the graphed points would lie on a straight line whose constant ratio of rise to run has the same value as the constant ratio in the recipe.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using ideas of times as much; using fractions; calculating percent given the part and the whole.

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

Answer

(1) 20:8, 10:4, and 5:2. **(2)** $\frac{5}{2}$ (or equivalent forms). **(3)** True.

(4) Approximately 70% (or a more precise value).

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.1; MP.2, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- In part (1) the ratio not circled, 5:7, isn't circled because it makes a different comparison, namely weekdays to total days. The value of that ratio is $\frac{5}{7}$, which is the fraction that appears in the true statement of part (3), "$\frac{5}{7}$ of the days that month were weekdays."

Extending the task

How might students drive the conversation further?

- Students might wonder if other months have the same ratios as February 2021. What is the ratio of weekdays to weekend days during the current month? What month(s) next year have the greatest percentage of weekend days?
- What is the percentage of weekend days for all of the next calendar year? (What would you estimate it to be?)

Related Math Milestones tasks

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

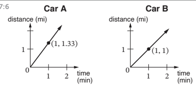
6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

Task **6:2 Prizes, Prices, and Percents** involves percent as a ratio per 100. Tasks **6:4 Gas Mileage** and **6:6 Planting Corn** involve proportional relationships.

7:6

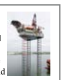
7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does that car move in 10 minutes?

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note, 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?



7:7

7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24 hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

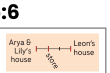
7:2

7:2 A utility pole 24 feet long has $28\frac{1}{4}$ inch circumference at the top and $47\frac{1}{8}$ inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

In later grades, tasks **7:6 Car A and Car B** and **7:8 Oil Business** involve proportional relationships. In task **7:7 Speed Limit**, two unit rates are compared, and in task **7:12 Temperature Change**, an average rate is calculated. In task **7:2 Utility Pole Scale Drawing**, lengths in the drawing have a constant ratio with lengths measured directly on the physical object.

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 l of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** involve the extension of multiplication and division from whole numbers to fractions, with attendant ideas of scaling.

Additional notes on the design of the task (continued)

- Task 6:10 was inspired by a home conversation with a sixth-grader who had opinions about what would be a fair ratio of school days to weekend days.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:10? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:10? In what specific ways do they differ from 6:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

