6:11 Area Expressions

Teacher Notes

Central math concepts

An expression records operations with numbers and with letters standing for numbers. To restate this in more active terms, one creates an expression by calculating with variables as if they were numbers. For example, suppose three items are purchased with prices *a*, *b*, and *c*, and suppose there is city, county, and state tax on the purchase, with respective tax rates given by *r*, *s*, and *t*. Then the total tax on the purchase could be calculated as $(r + s + t)\cdot(a + b + c)$.

A conceptually distinct way to figure the total tax on the purchase would be to compute the total tax on each item and sum those amounts: $r\cdot(a + b + c) + s\cdot(a + b + c) + t\cdot(a + b + c)$. The expressions $(r + s + t)\cdot(a + b + c)$ and $r\cdot(a + b + c) + s\cdot(a + b + c) + t\cdot(a + b + c)$ are equivalent, meaning they would have the same values no matter what values of the variables are substituted into them. This is intuitively plausible in the sales tax situation, and it's mathematically guaranteed by the distributive property. For that matter, the distributive property would apply even if some or all of "tax rates" were negative (this could conceivably happen if there were a rebate program of some kind in effect).

The properties of operations can be used not only to see that two given expressions are equivalent, but also more in a more active fashion to transform one given expression into another, equivalent expression—ideally an expression that is simpler than, or that adds insight to, the given one. For example, a faster than usual way to calculate the average of 84 and 28 would be to add 42 + 14; this method works because $\frac{(a+b)}{2} = \frac{1}{2}a + \frac{1}{2}b$. As another example, if $y = mx + b$ is a linear function with $b \neq 0$, then although *y* is not proportional to *x*, changes in *y* are proportional to changes in *x*, and this can be proved by applying properties of operations: if $y_1 = mx_1 + b$ and $y_2 = rx_2 + b$ are two different *y* values, then the change in quantity *y* equals

$$\begin{aligned} &y_2 - y_1 \\ &= (mx_2 + b) - (mx_1 + b) \\ &= mx_2 - mx_1 \\ &= m(x_2 - x_1) \end{aligned}$$

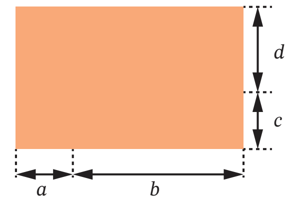
so that the change in *y* is always a constant multiple *m* of the change in *x*.

The transformations in task 6:11 centrally involve the distributive property, which states the mathematical relationship between multiplication and addition:

$$a(b + c) = ab + ac.$$

The distributive property is an identity, an equation that is true for all possible values of its variables. The left-hand side of the identity is a product of two terms (one of which is a sum), while the right-hand side of the identity is a sum of two terms (both of which are products). The distributive property allows us to rewrite sums as products and rewrite products as sums.

6:11 The diagram shows a rectangle. The variables *a*, *b*, *c*, and *d* are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

Answer

(1) Any three expressions equivalent to $(a + b)(c + d)$. Examples: $a(c + d) + b(c + d)$, $db + ca + ad + bc$, $(c + d)(b + a)$. The multiplication symbol × or * or · may be included or omitted. **(2)**, **(3)** Answers may vary. For example, applying the distributive property to $(a + b)(c + d)$ can result in $(a + b)c + (a + b)d$, which shows that these two expressions are equivalent. Another example: applying the distributive property to $ac + ad + bc + bd$ can result in $ac + ad + b(c + d)$, which shows that these two expressions are equivalent.

Click here for a student-facing version of the task.

Refer to the Standards

6.EE.A; MP.2, MP.3, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Many familiar techniques in algebra can be understood as applications of the distributive property:

- Distributing. When we distribute r in the example $r(k + p + 2) = rk + r(p + 2)$, we are applying the distributive property to rewrite a product of two terms as a sum of two terms. Note that we could apply the distributive property again to rewrite $r(p + 2)$ as $rp + 2r$. That would result in the identity $r(k + p + 2) = rk + rp + 2r$.
- Factoring. When we factor out y in the example $xy + yz = y(x + z)$, we are applying the distributive property to rewrite a sum of two terms as a product of two terms.
- Collecting like terms. When we collect like terms in the example $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = (\frac{1}{8} + -\frac{1}{2} + \frac{1}{4})x$, we are applying the distributive property to rewrite a sum of three terms as a product of two terms. Of course we could continue to evaluate the sum $\frac{1}{8} + -\frac{1}{2} + \frac{1}{4} = -\frac{1}{8}$, which would result in the identity $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = -\frac{1}{8}x$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: decomposing a rectangle into rectangles; applying area formulas and area reasoning; and viewing expressions as objects with structure.

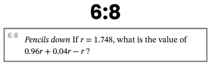
Extending the task

How might students drive the conversation further?

- Students could create two equivalent expressions for the perimeter of the rectangle and explain how one can be transformed into the other by applying properties of operations.
- If students know the triangle area formula $A = \frac{1}{2}b\,h$, and if they have used the trapezoid area formula $A = \frac{1}{2}(b_1 + b_2)h$, then they could apply the distributive property to the expression $\frac{1}{2}(b_1 + b_2)h$, and interpret the resulting expression as the sum of two triangle areas. Where are these triangles in the trapezoid? (Because a trapezoid can be decomposed into triangles, the trapezoid area formula isn't as important as the area triangle area formula.)

Related Math Milestones tasks

6:8


6:8 *Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r - r$?

Task **6:8 Evaluating an Expression** involves an expression that could be rewritten in a more convenient form by making use of its structure.

Additional notes on the design of the task

Area in the task could connect to area models students may have used for multi-digit multiplication. The partial-products calculation 45 × 63 = 40 × 60 + 40 × 3 + 5 × 60 + 5 × 3 corresponds to task 6:11 with a = 40, b = 5, c = 60, and d = 3.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:11? In what specific ways do they differ from 6:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.
(2) Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

In later grades, tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property. Task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

4:5

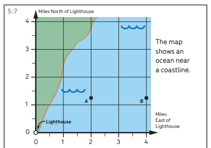
4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ____ (a)
$6 \times \frac{4}{7} =$ ____ (b)
$86 \times \frac{1}{86} =$ ____ (c)
$6 \times \frac{8}{2} =$ ____ (d)
$9 \times \frac{1}{9} =$ ____ (e)
$9 \times \frac{2}{9} =$ ____ (f)

(1g) Which answer is twice as much as the answer for (e)?
(1h) Which answer is six times as much as the answer for (a)?
(1i) Which two answers are equal?
(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

5:7

5:7



Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:14

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

3:10

3:10 Alice forgot what 7×8 equals. Alice knows that $5 \times 8 = 40$ and $2 \times 8 = 16$.
(1) Write a sentence to tell Alice how she can find the value of 7×8 by using the two facts she knows.
(2) Draw a diagram that could help Alice understand why your method works.
(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

In earlier grades, task **5:1 Juice Box Mixup** is a problem in context with an interpretation in terms of multiplication distributing over subtraction; tasks **4:5 Fraction Products and Properties**, **5:7 Shipwrecks**, and **5:14 Brandon's Equation** involve fraction products that could be evaluated using the distributive property; and task **3:10 Alice's Multiplication Fact** involves using the distributive property as a calculation strategy for a product of two one-digit numbers.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?