6:12 Coordinate Triangle

Teacher Notes

Math Milestones™

Central math concepts

After studying length as a measurable quantity in the primary grades, students in upper elementary grades learn to recognize area as a measurable attribute of plane figures. Area is the amount of two-dimensional surface a figure contains. Consistent with this idea, congruent figures are assumed to enclose equal areas. Upper-elementary grades students also learn the concepts involved in measuring area (CCSS 3.MD.C.5):

- **A unit of measure for area**: An area unit is built from a chosen length unit. Given a length unit, a square with side length equal to 1 unit, called "a unit square," is said to have "one square unit" of area.
- **Quantifying area**: A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Because a rectangle with whole-number side lengths of L units and W units can be tiled with an L-by-W array of unit squares, the area of such a rectangle equals the product $L \times W$. As students learn to multiply fractions, they understand that even when a rectangle has fractional side lengths of L units and W units, the area of the rectangle still equals the product $L \times W$.[†]

From these beginnings,

> Using the shape composition and decomposition skills acquired in earlier grades, students [in grade 6] learn to develop area formulas for parallelograms, then triangles. They learn how to address three different cases for triangles: a height that is a side of a right angle, a height that 'lies over the base' and a height that is outside the triangle.
>
> Through such activity, students learn that any side of a triangle can be considered as a base and the choice of base determines the height (thus, the base is not necessarily horizontal and the height is not always in the interior of the triangle). The ability to view a triangle as part of a parallelogram composed of two copies of that triangle and the understanding that area is additive ... provides a justification for halving the product of the base times the height, helping students guard against the common error of forgetting to take half.

Progression document, p. 19.[‡]

Triangles on the same base, and with the same height relative to that base, all have equal areas, even though their shapes look quite different. (See an animation illustrating this.) This principle, which is involved in part (2) of task 6:12, can be seen as a consequence of the formula $A = \frac{1}{2}\, b\, h$.

More directly, consider the first figure, in which parallelograms ABCD and ABEF are on the same base and between the same parallels. It can be shown using triangle congruence theorems that triangles ADF and BCE are congruent, so they have equal areas, and hence the area of ABED less the area of BCE equals the area of ABED less the area of ADF. Therefore the

> 6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

Answer

(1) 21 square units. **(2)** The area does not change.

Click here for a student-facing version of the task.

Refer to the Standards

6.G.A.1, 3; MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

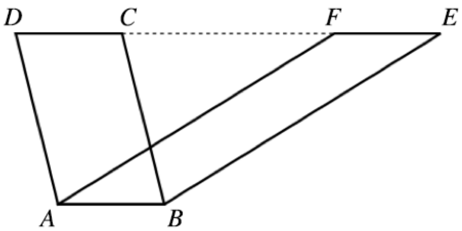
Aspect(s) of rigor:

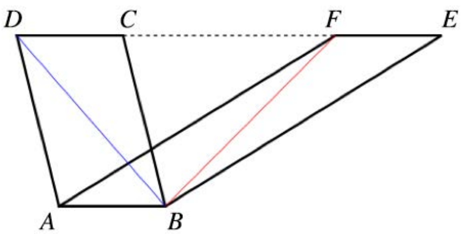
Concepts, Procedural skill and fluency

Additional notes on the design of the task

The task does not include a diagram, because mapping the given information into the coordinate setting is part of the work.

areas of the parallelograms are equal. And since the triangles ABD and ABF are half the parallelograms, the areas of the triangles on the same base and with equal heights are equal.





Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying by $\frac{1}{2}$; remembering a multiplication fact; graphing points on a coordinate plane; and understanding signed numbers on the coordinate plane.

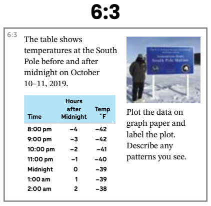
Extending the task

How might students drive the conversation further?

- Students could find a value of *x* such that the triangle with vertices (1, 2), (-5, 2), and (*x*, 9) is a right triangle.
- Students could find the coordinates of a point that makes a parallelogram with the given vertices of the triangle.

Related Math Milestones tasks

6:3

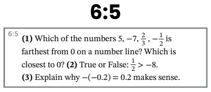


6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	−4	−42
9:00 pm	−3	−42
10:00 pm	−2	−41
11:00 pm	−1	−40
Midnight	0	−39
1:00 am	1	−39
2:00 am	2	−38

Plot the data on graph paper and label the plot. Describe any patterns you see.

6:5



6:5 **(1)** Which of the numbers 5, -7, $\frac{2}{3}$, $-\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

Task **6:3 South Pole Temperatures** involves positive and negative coordinates in the coordinate plane, and task **6:5 Positive and Negative Numbers** involves signed rational numbers on a number line.

Curriculum connection

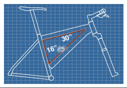
1. In which unit of your curriculum would you expect to find tasks like 6:12? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:12? In what specific ways do they differ from 6:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:10

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

8:3

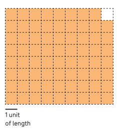
8:3 On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.



In later grades, task **7:10 Triangle Conditions** advances beyond area measurement to analyze length and angle measures in a triangle. Task **8:3 Bicycle Blueprint** involves the Pythagorean theorem, which among other things enables distances between two points to be calculated when the points don't share a common *x*-coordinate or a common *y*-coordinate.

3:3

3:3 **(1)** How much area is shaded?



(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: __________ Width: __________

4:13

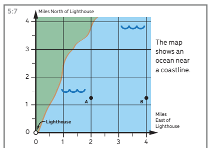
4:13 **(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

(2) A blue rectangle has length **1 ft** and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

5:7

5:7



Shipwrecks are at locations $A\,(2, 1\frac{1}{4})$ and $B\,(4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\,(4, 3\frac{1}{2})$ and $D\,(2, 3\frac{1}{2})$. **(1)** Mark *C* and *D* on the map and shade rectangle *ABCD*. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

In earlier grades, tasks **3:3 Length and Area Quantities** and **4:13 Area Units** involve concepts of area measurement. Task **5:7 Shipwrecks** involves the area of a rectangle specified by four points in the coordinate plane.

† An example from the NF *Progression* document: "Instead of using a unit square with a side length of 1 inch or 1 centimeter, fifth graders use a unit square with a side length that is a fractional unit. For example, a 5/3 by 1/2 rectangle can be tiled by 30 unit squares with side length 1/6. Because 36 of these unit squares tile a 1 by 1 square, each has area 1/36. So the area of the rectangle is 30 thirty-sixths, which is 5/3 × 1/2, the product of the side lengths." (p. 18)

‡ Common Core Standards Writing Team. (2013, September 19). Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?