6:13 Is There a Solution? (Multiplication)

Teacher Notes

Central math concepts

An equation can be viewed as a question: Which values from a specified set, if any, make the equation true? Solving an equation is a process of reasoning resulting in a complete answer to that question.

Task 6:13 explores the extension of the number system from whole numbers to fractions. If fractions didn't exist, we would have to invent them. Consider the dilemma of a third-grade student who solves problems like $72 \div 9 = 8$ but who may wonder what to make of a problem like $72 \div 10 = ?$. The unknown factor can't be 7 (because $10 \times 7 = 70$, which is too small); and the unknown factor can't be 8 (because $10 \times 8 = 80$, which is too large). So maybe the unknown factor is between 7 and 8? Closer to 7, because 70 is closer to 72 than 80 is? It isn't until the upper elementary grades that students fully integrate positive fractions into their expanding system of numbers and operations, and find solutions to equations like $72 \div 10 = ?$, or indeed $241p = \frac{3}{4}$.

The questions in task 6:13 could be settled procedurally by dividing both sides of the equation by 241 and then calculating the value of the quotient $\frac{3}{4} \div 241 = \frac{3}{964}$. However, the task does not ask for the exact value of p. The task thereby aims at non-procedural skills, especially the algebraic skill of looking for and making use of structure (CCSS MP.7), which grows in importance throughout students' study of algebra. And the task prioritizes non-procedural knowledge, like knowing how the size of a product depends on the size of its factors. If we can view the equation $241p = \frac{3}{4}$ as asking a question, the question might read like this: "I am thinking of a number. 241 times as much as my number equals $\frac{3}{4}$. What is my number?" This rendering of the equation emphasizes the sizes of the quantities involved and the meaning of the operation involved.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: the written conventions of algebra, specifically the conventional omission of the multiplication symbol in a product; understanding of how the size of a product depends on the size of its factors; understanding division as an unknown factor problem; and solving one-step equations.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $ap = b$ that has no solution even among the fractions. (Do not assume that a and b are necessarily whole numbers, but assume that a is nonzero.) As part of this discussion,

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Answer

There is no whole number that solves the equation. There is a non-whole number that solves the equation. (Reasoning for these decisions may vary.)

Click here for a student-facing version of the task.

Refer to the Standards

6.EE.B.5; MP.1, MP.3, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- The intent of saying "pencils down" is to invite a conceptual approach.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:13? In what specific ways do they differ from 6:13?

students could consider what happens when both sides of the equation are multiplied by $\frac{1}{a}$. (Recall from the properties of operations that for every nonzero number a, there exists a number $\frac{1}{a}$ such that $\frac{1}{a} \times a = 1$. Recall too from the properties of operations that $1 \times q = q$ for every number q.)

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $ax = b$ that has two different solutions.
- Students could make sense of the equation $241p = \frac{3}{4}$ by creating word problems in which the answer is the solution to the equation. (For example, "A stack of 241 sheets of paper was measured to be $\frac{3}{4}$ in thick. How thick, measured in inches, is one sheet of paper?")

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

5:2

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

Task **6:1 Charging Cord** is a word problem for which a natural equation model ($\frac{2}{3}x = \frac{1}{2}$) also involves a product that is less than the coefficient.

In later grades, task **7:5 Is There a Solution? (Addition)** involves the next major extension of the number system, from the fractions to the rational numbers.

In earlier grades, task **5:2 Water Relief** involves the division of two whole numbers leading to a fractional quotient.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?