6:14 Dividing Decimals and Fractions

Teacher Notes

Central math concepts

Task 6:14 focuses on procedures. The two types of problems considered—calculating the quotient of two decimals, and calculating the quotient of two fractions—frequently arise in the course of solving problems. For both problem types, an efficient pencil-and-paper algorithm exists.

> **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
>
> **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Algorithms are usefully distinguished from strategies (CCSS, p. 85; see figure). Strategies are "purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another." Mental calculation often uses strategies. For example, we could calculate 15 × 12 by thinking of 3 × (5 × 12) = 3 × 60, or by thinking of 10 × 12 + 5 × 12, or in numerous other ways. As another example, we could calculate $\frac{2}{3} \div \frac{3}{4}$ by thinking of $\frac{8}{12} \div \frac{9}{12}$ and thinking in units of twelfths to see that $\frac{8}{12} \div \frac{9}{12}$ is 8 ÷ 9 or $\frac{8}{9}$. Flexible procedural fluency based on place value, properties of operations, relationships between operations, and ideas of equivalence is valuable both in practical terms and also as a way of doing arithmetic that prefigures algebra. In algebra, strategies are the most common way of working.

Algorithms are inflexible by definition. One step follows another in the prescribed order. Some algorithms are also much simpler to execute than others. The standard multi-digit addition algorithm is less complex than the standard multi-digit subtraction algorithm. The standard algorithm for dividing fractions is less complex than the standard algorithm for dividing multi-digit numbers.

An important value in mathematics education is that of being able to solve problems in multiple ways. This brings the pleasures of seeing how a coherent subject holds together, and it allows students to check answers, unify their understanding of concepts, and learn from different ways of thinking that emerge in the classroom community. A parallel but also important outcome of mathematics education is for students to be supported in gaining procedural fluency with algorithms for the actually quite small set of recurrent problem types for which an algorithm exists. (This small set can be found in the CCSS-M by searching for "algorithm.") With division of fractions and decimals in grade 6, students who began learning to say the counting words in kindergarten reach the culmination of many interwoven learning progressions in the procedures for adding, subtracting, multiplying, and dividing whole numbers, fractions, and decimals.

> 6:14 *Pencil and paper* **(1)** 81.53 ÷ 3.1 = ?
> **(2)** $\frac{7}{8} \div \frac{2}{3}$ = ? **(3)** Check both of your answers by multiplying.

Answer

(1) 26.3. **(2)** $\frac{21}{16}$. **(3)** See image.

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   26.3
 ×  3.1
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    263
   7890
 ------
  81.53
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$$\frac{2}{3} \times \frac{21}{16} = \frac{2 \times 21}{3 \times 16} = \frac{42}{48} = \frac{7}{8}$$

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.NS.B; MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

Checking the answers to parts (1) and (2) by multiplying offers additional procedural practice and reinforces the relationship between multiplication and division: *C* ÷ *A* is the unknown factor in *A* × ☐ = *C*.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value, number sense of decimals, and decimal notation; number sense of fractions; multiplication facts and related quotients; and the relationship between multiplication and division.

Extending the task

How might students drive the conversation further?

- Students could make sense of their quotients another way by making estimates of the values; for example, 81.53 ÷ 3.1 ≈ 81 ÷ 3 = 27 which compares well to 26.3, or $\frac{7}{8} \div \frac{2}{3} \approx 1 \div \frac{2}{3} = \frac{3}{2}$ which compares well to $\frac{21}{16}$ because $\frac{3}{2} = \frac{21}{14}$.
- Students could enter $(\frac{7}{8})/(\frac{2}{3})$ into a calculator, obtaining the result 1.3125. Use pencil and paper to multiply this number by 16. What is the result? Does the result make sense?

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.
(2) Show calculation 1(a) on a number line.

8:6

8:6 Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

5:5

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10$ = ?	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3}$ = ?	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8}$ = ?	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5}$ = ?	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5)$ = ?	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

Tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** involve fraction quotients in context. Task **6:13 Is There a Solution? (Multiplication)** considers an unknown factor problem as an algebraic equation.

In later grades, task **7:9 Calculating with Rational Numbers** involves extending fraction procedures to the rational number system. Task **8:6 Rational Form** (part (b)) involves fraction calculations expressed in the notation of positive and negative exponents.

In earlier grades, task **5:5 Calculating** includes a range of procedural tasks, some perhaps more amenable to strategies and others perhaps more amenable to algorithms.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:14? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 6:14? In what specific ways do they differ from 6:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?