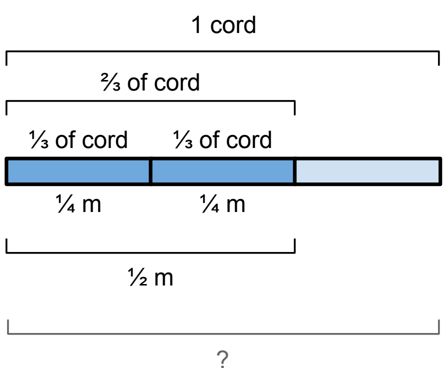
6:1 Charging Cord

Teacher Notes

Math Milestones™

Central math concepts

Unit fractions play a role, visible or invisible, in most fraction situations. For example, suppose we knew that $\frac{1}{3}$ of a cup of flour weighs $4\frac{1}{4}$ ounces. Then we would know that 1 cup of flour weighs 3 times as much. Or, suppose we knew that in a newspaper survey, $\frac{1}{10}$ of the people surveyed, or 85 people, preferred a certain monthly subscription plan. Then we would know that 850 people were surveyed. Even when a problem is given without unit fractions, thinking about unit fractions in the problem can help. For example, if $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ m long, then how long in meters is $\frac{1}{3}$ of the cord? Half as long, or $\frac{1}{4}$ m. The realization that $\frac{1}{3}$ of the cord is $\frac{1}{4}$ m long may be helpful in determining the full length of the cord. (See the diagram.)



Fraction equivalence also plays a visible or invisible role in many fraction problems. For example, if we replace the fraction $\frac{1}{2}$ in task 6:1 with the equivalent fraction $\frac{2}{4}$, then the original statement that $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ m long becomes the equivalent statement that **2 thirds** of a charging cord is **2 fourths** of a meter long. Thirds of the cord match up with fourths of a meter, as shown in the diagram. So 1 cord is 3 thirds which is 3 fourths of a meter.

Algebraically, the structure of task 6:1 corresponds to the equation $\frac{2}{3}L = \frac{1}{2}$, where L is the length of the cord in meters. Dividing both sides of the equation by $\frac{2}{3}$ produces the length of the cord: $L = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$. This process could correspond to an understanding of division along the lines of, "$\frac{2}{3}$ times something is $\frac{1}{2}$...so the something must be $\frac{1}{2} \div \frac{2}{3}$." This is a generalization of the reasoning that students used in grade 3, when they

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

Answer

$\frac{3}{4}$ m. (The decimal 0.75 m is also correct.)

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.EE.B.7; MP.2, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Task 6:1 could be stated in equivalent terms as, "$\frac{2}{3}$ of a charging cord is 50 centimeters long. How long is the cord?" The structure of this form of the problem is $\frac{2}{3}C = 50$, where C is the length of the cord in centimeters. In this form, the problem continues to reward thinking along the lines discussed in the Central Math Concepts. For example, dividing both sides of the equation by $\frac{2}{3}$ gives $C = 50 \div \frac{2}{3} = 50 \times \frac{3}{2} = 75$; and thinking that $\frac{1}{3}$ of the cord is 25 cm long may suggest that the cord is 3 × 25 cm long. However, when providing the answer to the task, the length of the cord must be expressed in meters.

solved problems like "7 times something is 42, so the something must be 42 ÷ 7."

Calculating the quotient $\frac{1}{2} \div \frac{2}{3}$ is equivalent to calculating the product $\frac{1}{2} \times \frac{3}{2}$. This circumstance is connected to a second way to solve the equation $\frac{2}{3}L = \frac{1}{2}$, which is to multiply both sides of the equation by $\frac{3}{2}$:

$$\frac{3}{2}\left(\frac{2}{3}L\right) = \frac{3}{2}\left(\frac{1}{2}\right)$$

$$\left(\frac{3}{2} \cdot \frac{2}{3}\right)L = \frac{3}{2}\left(\frac{1}{2}\right)$$

$$(1)L = \frac{3}{4}$$

$$L = \frac{3}{4}.$$

Multiplying both sides of the equation by $\frac{3}{2}$ corresponds to the idea that "I could turn $\frac{2}{3}$ into 1 whole by multiplying it by one-and-a-half, so the length of the cord must be one-and-a-half times $\frac{1}{2}$ m."

For an illuminating discussion of fraction division with numerous diagrams and examples, see the 2017 series of blog posts by William McCallum and Kristin Umland on MathematicalMusings.org (part 1, part 2, part 3, part 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about unit fractions in context; using ideas of scaling and times-as-much; and defining a variable to create and solve a constraint equation.

Extending the task

How might students drive the conversation further?

- Students who worked with units of centimeters and students who worked with units of meters could relate the two approaches by creating a combined diagram that aligns the centimeter diagram and the meter diagram.
- Students could discuss whether it makes sense that the cord is less than 1 meter long, in light of the fact that more than half of the cord is only half of a meter.

Additional notes on the design of the task (continued)

- Because length is a continuous quantity, length quantities can be good candidates for fraction contexts. Length has special importance because length is the basic metaphor for all measured magnitudes.
- Task 6:1 isn't well thought of as a proportional relationships problem. The context doesn't feature two variable quantities, just a single unknown value. Thus, $\frac{2}{3}L = \frac{1}{2}$ or $\frac{2}{3}C = 50$ are constraint equations, not function equations. As in previous grades, the division operation here simply finds an unknown factor; it doesn't yield a third kind of quantity, as when we divide distance by time to create a new quantity speed, or divide mass by volume to create a new quantity density. (That said, fraction quotients and fraction equivalence can sometimes involve versions of proportional thinking, as for example when we think, "If the parts are twice as many, each part must be half the size.")

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:1? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:1? In what specific ways do they differ from 6:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3) (a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:14

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Like task 6:1, task **6:9 Truckload of Gravel** is a word problem with an unknown factor that is a quotient of fractions. Task **6:13 Is There a Solution? (Multiplication)** deals with the existence and nature of solutions of a particular equation with form $ax = b$. Task **6:2 Prizes, Prices, and Percents** includes percent problems with the mathematical structure of an unknown factor problem (parts (2) and (3)). Task **6:4 Gas Mileage** includes several scaling problems in the context of a single proportional relationship. Task **6:14 Dividing Decimals and Fractions** is a procedural task that involves dividing fractions.

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

In later grades, task **7:13 Wire Circle** is a word problem that could be solved by creating a constraint equation of the form $px + q = r$, where p, q, r, and x are all non-whole numbers.

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

In earlier grades, task **5:13 Frozen Yogurt Machine** is a multi-step word problem involving multiplication and division with unit fractions. Task **5:1 Juice Box Mixup** involves an unknown factor that is a fractional quotient of whole numbers. Task **5:5 Calculating** is a procedural task that includes quotients involving whole numbers and unit fractions.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?