6:2 Prizes, Prices, and Percents

Teacher Notes

Math Milestones™

Central math concepts

One percent of a quantity means $\frac{1}{100}$ of the quantity. And because p% of a quantity is p times as much as 1% of a quantity, it follows that p% of a quantity is $\frac{p}{100}$ of the quantity.

Unit fractions with denominators 2, 4, 5, 10, 25, and 50 have convenient equivalent fractions with denominators of 100:

$\frac{1}{2} = \frac{50}{100}$ $\frac{1}{4} = \frac{25}{100}$ $\frac{1}{5} = \frac{20}{100}$ $\frac{1}{10} = \frac{10}{100}$ $\frac{1}{25} = \frac{4}{100}$ $\frac{1}{50} = \frac{2}{100}$

These fraction equivalences imply useful fraction-percent equivalents:

$$\frac{1}{2} \text{ of } = 50\% \text{ of}$$
$$\frac{1}{4} \text{ of } = 25\% \text{ of}$$
$$\frac{1}{5} \text{ of } = 20\% \text{ of}$$
$$\frac{1}{10} \text{ of } = 10\% \text{ of}$$
$$\frac{1}{25} \text{ of } = 4\% \text{ of}$$
$$\frac{1}{50} \text{ of } = 2\% \text{ of}$$

From these unit fraction equivalents, other equivalents can be found by multiplying. For example, knowing that $\frac{1}{5}$ of a quantity is 20% of the quantity, one also knows that $\frac{4}{5}$ of a quantity is 80% of the quantity.

The correspondence between decimals and percents arises by writing fractions with denominators of 100 in decimal notation; for example, $\frac{1}{100} = 0.01$, so that 1% of a quantity is 0.01 times the quantity. To say it another way, 1% of a quantity means 0.01 of each unit of the quantity.

Writing percents as decimals allows base-ten algorithms to be used for calculations, as when we calculate 75% of a \$50 prize by multiplying 0.75 × 50 = 37.5 (see part (1) of task 6:2). But even without doing any base-ten calculations, it could be argued that 75% of a \$50 prize is preferable to 33% of a \$100 prize, because although the prize is half as small, the percent is more than twice as large. Generalizing this analysis, if we are choosing between a% of a small prize, \$$S$, and b% of a large prize, \$$L$, then the condition for being indifferent to the choice is when $\frac{a}{100} \times S = \frac{b}{100} \times L$. The equation for the condition of being indifferent to the choice could be written equivalently in different forms, such as $\frac{a}{b} = \frac{L}{S}$, in which form the equation states the condition that the ratio of the percents equals the inverse ratio of the prizes.

6:2 **(1)** Would you prefer 33% of a \$100 prize or 75% of a \$50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a \$16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a \$100 item adds ___ dollars to the cost. A 3% tax on a \$1 item adds ___ dollars to the cost.

Answer

(1) 75% of a \$50 prize. **(2)** 32. **(3)** 7%. **(4)** 0.0625, 1/16. **(5)** \$17. **(6)** 3, 0.03.

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.3c; MP.2, MP.4, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- Parts (4) and (5) are connected through the value 0.0625 = 1/16 and the price of \$16.
- The second sentence of part (6) is designed to connect to the idea that 1% of a quantity means 0.01 for each 1 unit of the quantity. (So, 3% of a quantity means 0.03 for each 1 unit of the quantity, the units here being dollars.)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental calculation; using ideas of times as much; using fractions and fraction equivalence; using decimals; and using fraction-decimal equivalents.

Extending the task

How might students drive the conversation further?

- Students could find the percent of a $50 prize that is worth the same as 33% of a $100 prize.
- By using a calculator to try combinations, students could create versions of part (1) that are "close calls," like 53% of a $43 prize vs. 24% of a $93 prize.

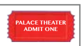
Related Math Milestones tasks

6:10

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

7:11

7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of $7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*



7:1

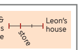
7:1 The cost of a phone is the phone's price, $264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:4

7:4 *"Foul Play."* The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot. His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 one-point free throws. The defender chose to foul and later wondered if it was a good choice. **(1)** To analyze the defender's choice, let's assume that for the Pistons player, every 3-point shot has probability $\frac{1}{3}$ of going in, and every free throw has probability 90% of going in. **(a)** If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. **(2)** Write a paragraph arguing for or against the defender's choice, based on probability calculations and/or simulations.

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Task **6:10 Weekdays and Weekend Days** involves ratios, fractions, and percent.

In later grades, task **7:11 Ticket Offers** compares a percent discount to a discount in absolute dollars. Task **7:1 Phone Cost** uses percent in relation to the distributive property and algebraic expressions. Task **7:4 Foul Play** involves a probability given as a percent.

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** involve the extension of multiplication and division from whole numbers to fractions, with attendant ideas of scaling.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:2? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:2? In what specific ways do they differ from 6:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

6:2 Prizes, Prices, and Percents

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?