6:4 Gas Mileage

Teacher Notes

Central math concepts

This task is a deep dive into a situation in which two quantities are in a proportional relationship with one another. In fact, there are two proportional relationships in the situation:

1. The **number of miles** the narrator can drive is proportional to the **number of gallons** available. In other words, there is a fixed number e such that the number of miles driven is e times the number of gallons used. The number e is the unit rate for this relationship.
2. The **number of gallons** the narrator will use is proportional to the **number of miles** driven. In other words, there is some fixed number r such that the number of gallons used is r times the number of miles driven. The number r is the unit rate for this relationship.

The different subparts of the task concern a range of concepts involved in proportional relationships, including (a) using multiplicative scaling to find an unknown quantity in a situation with a proportional relationship; (b) calculating unit rates; (c) using unit rates to find an unknown quantity in a situation with a proportional relationship; and (d) graphing a proportional relationship. The graph serves as a single picture of all the possible covarying values in the relationship.

For part (1), the values have been chosen so as to facilitate mental calculation using scaling ideas as an approach to finding an unknown quantity in a proportional relationship. For example, 57 is one-tenth of 570, and if we are driving one-tenth as far, we will use one-tenth as much gas. Meanwhile, parts (2) and (3) are designed to promote the approach of multiplying by the unit rate as an approach to finding an unknown quantity in a proportional relationship. For example, with 11 gal of gas, the number of miles we can drive can be found by using the unit rate of 38 miles per gallon: $38 \cdot 11 = 418$. In general then, the task is designed to promote the use of scaling and unit rates over the approach of setting up a proportion and cross-multiplying.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task:

- Mental calculation.
- Using units as guideposts to thinking about a problem.
- Thinking about measurement scales on the axes of a coordinate plane.
- Using ideas of scaling and times-as-much to multiply.

Extending the task

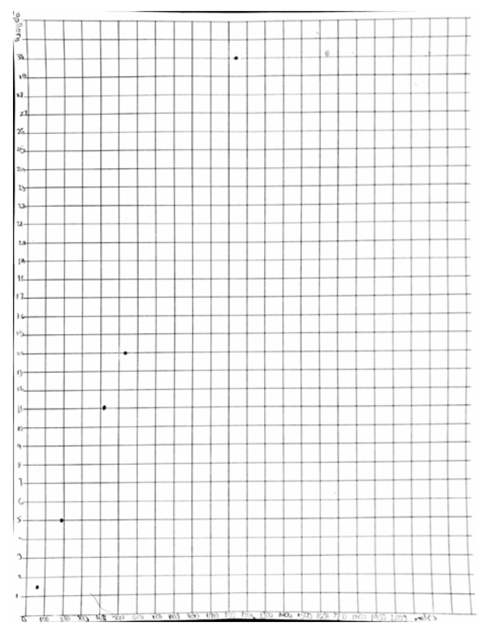
How might students drive the conversation further?

6:4

My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Answer

(1) **(a)** 1.5. **(b)** 150. **(c)** 190. **(d)** 1,140. **(2)** 38 miles per gallon and 0.026 gallon per mile. For the second unit rate, rounded values such as 0.03 gallon per mile are also correct, and the fraction form 1/38 gallon per mile is also correct. **(3)** **(a)** 14. **(b)** 418. **(4)** See example table and example graph.



Note: The axes could be interchanged and the graph would still be correct.

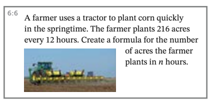
Miles driven	Gallons of gas used
57	1.5
190	5
1,140	30
532	14
418	11

Note: The left and right columns could be interchanged and the table would

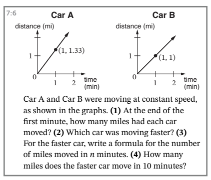
- If students did not answer part (3) by multiplying by the appropriate unit rate found in part (2), they could try that approach and verify that it leads to the same answers they obtained by other means.
- By attending to precision in creating the graph, students may notice that their plotted points appear to fall on a straight line. Students could conjecture that this will be the case for any graph of a proportional relationship. Instead of immediately receiving a definitive answer to their question, students could be told that this important question will become a focus of their work on proportional relationships in grades 7 and 8.
- For each pair of values in their table, students could calculate both possible quotients. Every row should yield the same two quotients, which are the two unit rates. Students might notice that the two constants are reciprocals of one another, and a mathematical discussion could aim for an explanation of why that is so.

Related Math Milestones tasks

6:6

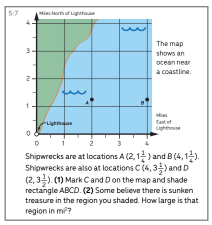

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

7:6



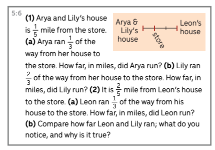
7:6 Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

5:7



5:7 Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:6



5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

Another task for Grade 6 that prominently features a proportional relationship is **6:6 Planting Corn**.

In later grades, task **7:6 Car A and Car B** prominently features a proportional relationship.

In earlier grades, task **5:7 Shipwrecks** features the coordinate plane and Math Milestones task **5:6 Corner Store** features the multiplicative scaling idea of times-as-much.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

still be correct. Also, any order of the rows is correct.

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.2, 3; MP.1, MP.2, MP.4, MP.5, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The task does not provide a pre-labeled coordinate grid, because thinking about what the labels ought to be is part of the work identifying the quantities in the situation that are in a proportional relationship.
- Students are not expected to know that the points they plot should fall along a straight line; the task is designed to make this observation a potential natural outgrowth of working on the task.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:4? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:4? In what specific ways do they differ?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?